Problem Set 1: Fibonacci

Opens: 3 p.m. October 4th, 2021

Due: 3 p.m. October 18th, 2021

- You must work independently.
- Write your solutions clearly and show all of your work.
- Include your name, student ID number, and email address.
- Email a pdf file of your solution to `ugrad_services@math.utah.edu` by the deadline.
- A winner will be decided on the basis of the best solution submitted. If no best solution can be determined (i.e. there exist relatively identical solutions), the winner will be the student who submitted their solution first.
- Each submission will be given 3 points for a fully correct solution and 1-2 points for a partially correct solution. The winner of each problem set will get a bonus of ϵ points.
- Please don't just search online for a solution - that isn't the point of this contest.
- Enjoy the problems!

Recall that the *Fibonacci Sequence* F_n is defined by:

$$F_0 = 0, F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2$$

Problem 1 [1 point]: Prove that the Fibonacci sequence satisfies

$$F_{2n-1} - 1 = F_{2n-2} + F_{2n-4} + \cdots + F_0 \text{ for } n \geq 1$$

Problem 2 [1 point]: Show that all positive integers can be written as a sum of distinct terms of the Fibonacci sequence.

Problem 3 [1 point]: Prove that for any positive integer k, there exists some $n \geq 1$ such that F_n is divisible by k.