AN INTRODUCTION TO CREDIT MIGRATION MODELING

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Introduction

Credit risk management has become one the most popular research areas in mathematical finance. It has been brought to importance by the numerous corporate bankruptcies that plagued the United States' economy after the internet stock bust late in the twentieth century. During that time, many billions of dollars were lost by major investors who had a poor understanding of credit risk in the market. Part of this failure was rooted in a misunderstanding in the credit ratings that were assigned by the companies Standard and Poor's (S&P) and Moody's Investor Service (Moody's). These firms give a letter grade representing the credit worthiness to various corporations. If a specific company is perceived to be in excellent financial shape, it will be given a "AAA" rating. If it is very close to defaulting on its debt, it will receive a "CCC" rating through S&P and a "Caa" rating through Moody's. To establish a finer level of rating, the intermediate states AA, A, BBB, BB, and B along with +/- to each of these divisions are allowed (note that this is S&P's terminology an that Moody's has a similar system with different notation).

Each day, companies that have active bonds (publicly traded debt issued to raise capital which terminates as soon as the principle and interest are paid) are evaluated to determine their credit worthiness. If something alters a company's ability to pay off debt, their rating will shift higher if it is believed that their financial position has strengthened or lower if their financial position has weaken (these changes are called credit migrations or credit transitions). Therefore, credit ratings indicate a company's financial strength, but do not explicitly predict the probability of migrating into another rating. This issue is resolved through credit migration modeling.

The usefulness of credit migration models has become essential to risk management. The New Basel Accord is an international agreement adopted by all United States banking institutions that has put stricter restraints on the amount of credit risk banks are allowed to take. Part of this agreement requires banks to use credit migration models that are subjected to less uncertainty to reduce unnecessary risk. In addition, all modern credit derivative pricing models (a financial instrument that transfers the risk associated with holding an asset without being transferred) rely on the probabilities estimated by credit migration models to generate fair pricing. Therefore, developing models that are as accurate as possible is an essential task of risk management.

Since I am just beginning my research, I have spent my time gathering information about the work that is previously done to set the stage for original research in the future. My writing is geared toward novices in mathematics.

Literature Review

The first credit migration model to be based on observable data was developed by Jarrow *et al* (1997) as a discrete-time homogenous Markov chain (note that it added D as a default state which is not found in either Moody's or S&P's notation). A Markov chain is a series of probabilities that are organized into a rectangular array called a "matrix".

Each cell is called a "transition probability" because a cell in the ith row and jth column of the matrix represents the probability of transitioning to the jth state when currently in the ith state where i and j are whole numbers that range from one to the total number of states allowed in the model (usually eight). In other words, the probability of migrating to a AA state when currently in a AAA state is found in the cell that occupies the first row and second column of the matrix. The probabilities are calculated by dividing the number of companies that have moved from one state to another by the total number of companies in question for all permutations. These numbers represent the frequency of a particular migration. Statistical theory tells us that with enough observations, the frequency of an event will converge to its probability (this principle is known as the "Strong Law of Large Numbers"). This theorem has an intuitive analog: the number of times a penny lands on heads after repetitive flips will eventually average out to be one-half—the probability of getting heads in the first place.

In addition to formulating a discrete-time homogenous Markov chain, Jarrow *et al* (1997) show that a continuous-time model can be constructed with more complicated mathematics. When developing a continuous-time Markov chain, a practitioner is required to estimate the values of a "generator matrix" using a "maximum-likelihood" estimation technique. A generator matrix can be thought of as the backbone to a transition matrix because it allows for a systematic calculation of the transition matrices for any period of time once it is computed. Yet still, given an estimated generator matrix, it is not guaranteed that a corresponding transition matrix will exist. This problem is presented in research by Israel *et al* (2001). They conclude that existence is guaranteed if all of the elements off the diagonal of the generator matrix are strictly positive; if the computed transition matrix does not have a zero in any entry; and if all rows of the generator matrix must sum to zero. Further work by Lando and Skødeberg (2002) determined that existence is not guaranteed, but nearly achieved. This implies that the existence problem will not be an issue in most instances.

Lando and Skødeberg (2002) were the first to criticize the use of a discrete-time model in favor of a continuous-time process. First, they note using a least-likelihood estimator for the transition probabilities will produce the lowest expected error under reasonable assumptions. This is because transitions that jump many states are infrequent events. Consequently, the Strong Law of Large Numbers will not apply because there are not enough observations to ensure convergence to the true probabilities. Therefore, it is nearly guaranteed that frequency based estimation found in discrete-time approaches will produce worse estimates than that of log-likelihood estimation. Another advantage of the continuous-time approach is that it greatly reduces the number of events with zero probability—an essential improvement since even an extremely rare event is always possible. It achieves this by counting both direct and indirect migrations.

In a direct migration, a transition will take place from the present state to another without any other migrations. In an indirect migration, a plurality of movements will occur that will eventually reach a specific state. For example, if a firm moves from AA to B to CC to D in a year, then there would be one direct migration from AA to B, from B to CC and from CC to D. In an indirect framework, all of the direct migrations would be recorded in addition to all combinational transitions such as from AA to B, from AA to CC, etc. Lando and Skødeberg (2002) claim that using this procedure helps minimize the downward migration bias created by successive downgrades right before a firm defaults.

Christensen *et al* (2004) found more supporting research for a continuous-time model. They conclude that it produces a tighter confidence interval than that of a discrete-time model. This implies that the transition probabilities can be used to take a more aggressive position with an asset without incurring more risk.

Still, there are many that have found problems with the homogeneity assumption. Bangia *et al* (2000), Nickell *et al* (2000) and Kavvathas (2001) have all found that credit migrations are dependent upon the business cycle. This implies that as the state of the economy changes from a rising to a lowering market, the transition probabilities will change as well. Therefore, assuming homogeneity will not be a good idea over long periods of time because the transition probabilities would have changed during the long timeframe. In anticipation of dealing with this problem, Lando and Skødeberg (2002) included a continuous-time inhomogeneous Markov chain that uses the Aalen–Johansen estimator to form the generator matrix. They note that in this case, this estimator will have a lower expected error than any other estimator.

Jafry and Schuermann (2004) developed a systematic way to compare different models and found that while the advantages of using a continuous-time model over a discrete-time model were very large (15-30% gain), the advantages gained by using a non-homogenous over a homogenous model were slight (less than 2% gain) under all market conditions. Therefore, it can be concluded that since it takes 100 times more computing power to calculate an Aalen–Johansen estimator than a maximum-likelihood estimator, under most circumstances the homogenous estimator should be used if the extra calculation time is not justified.

The next step in this research was undertaken by Frydman and Schuermann (2005) who developed what they refer to as a "Markov mixture model", which weights time to account for different effects. They were able to do this by constructing two transition matrices for a given time period—one that is defined up until the current date and another that is defined in the past. For example, when forming a one-year Markov mixture model, a one-year transition matrix would be calculated in the timeframe from today's date to a year ago and another from a year ago to two years ago. The two matrices are then weighted so that their combined influence is one when added together. In turn, this will create a new transition matrix (which is no longer Markovian) that contains past information as well as present. In theory, this model can be calibrated to accommodate business cycle effects by adjusting the weights toward or away from the past (although it is not done in their paper).

Conclusion

It has been shown that credit migration modeling is an important part of the world's economy through its role in risk management. Over the last few years, continuous-time models have phased out discrete-time models because their results better mirror real-world expectations. Still, no model correctly takes into effect the role of the business cycle even though many have shown that it has a strong role to play in influencing transition probabilities. The purpose of my future research will be to make the past information relevant by expanding on the work of Frydman and Schuermann (2005).

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