VIGRE Research Proposal

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Time Delays in Differential Equations Modeling Traffic Flow

Many differential equations are useful in modeling natural phenomenon. An ordinary differential equation, such as the exponential population growth model $P'(t)= a P(t)$ can be used to model the growth or decay of a population. However, such a model does not take into account the effects of time delays such as the time for the animal to reach it's reproductive stage in the life cycle. A time delay alters a differential equation by making the rate of change of a variable depend not only on it's current value. but also on it's value at a previous time. This effect can dramatically change the solution to the differential equation. For example, $P'(t) = a P(t-b)$ where b represents time delay can exhibit oscillations unlike the original ODE. This shows that time delays can cause qualitative differences in solutions.

Bando, Hasebe, Nakanishi, and Nakayama [PRE 51, 1035 (1995)] proposed a differential equation model for traffic known as the "optimal velocity model" (OVM)

$$x_n''(t) = a\{ V(x_{n+1}(t) - x_n(t)) - x'_n(t)\}$$

Here, x_n represents the position of each car. V is a function that determines the optimal velocity. And 'a' is a measure of the time it takes for the car to accelerate.

The driver of the nth car follows the (n+1)th car. The driver of a car would determine an optimal velocity V based on the positions of both cars given by $x_{n+1}(t) - x_n(t)$. Realistically, a difference will exist between the actual velocity, $x_n'(t)$, and the optimal velocity of a car, and the driver will respond to this with an appropriate acceleration. The coefficient 'a' here represents the sensitivity of the driver.

Bando, et al. added a single time delay to the optimal velocity model for another study [PRE 58, 5429 (1998]. They analyzed the system with time delay through a linear analysis and numerical simulations. A small time delay resulted in the solutions behaving as if there was no time delay at all. However, as the time delay is increased, a small overshoot emerged in the numerical simulations. The practical results of such time delays can be seen in everyday traffic patterns, such as the periodic slowing down of cars on a freeway.

This project sets out to continue the study of time delays in the OVM. First, to learn about the basics of differential equations, we plan to reproduce the results of Bando, et al.

Then, a linear stability analysis will be made on the effects of adding multiple time delays and/or a spread of time delays in numerical simulations. We will also study how the nth car follows along a road with periodically placed stop signs or stoplights.

Finally, a study will be made as to whether the time delay numbers used by Bando, et al. were too high or too low. We will look specifically for interesting phenomenon at physically reasonable time delays.

The motivation for this project is to better understand how traffic flow can be modeled so that a computer program could later be developed that would model traffic flow based on acquired data. This will be accomplished through the study of how time delays affect the optimal velocity model ODE, and analyzing how the model could be used effectively for traffic problems.