



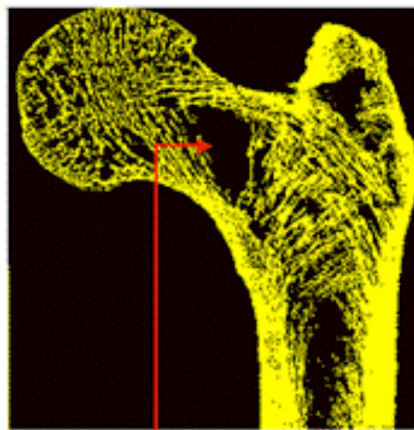
Correlation Functions and Osteoporotic Bone Structure

Casandra Hutchinson

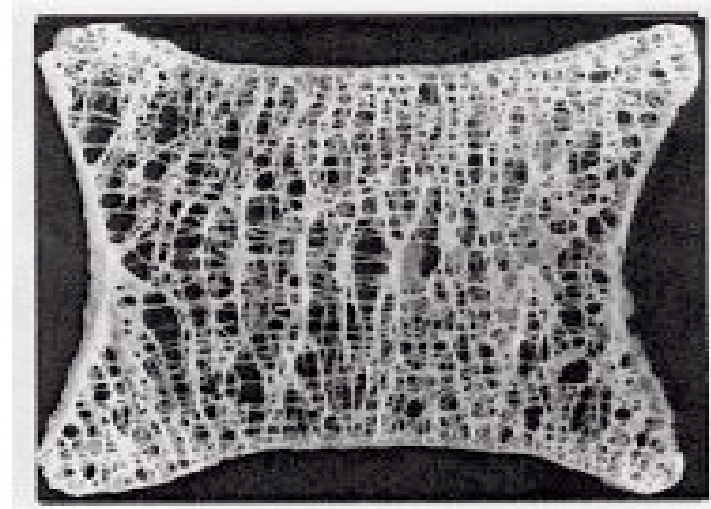
Under Instruction from Elena Cherkaev

Osteoporosis

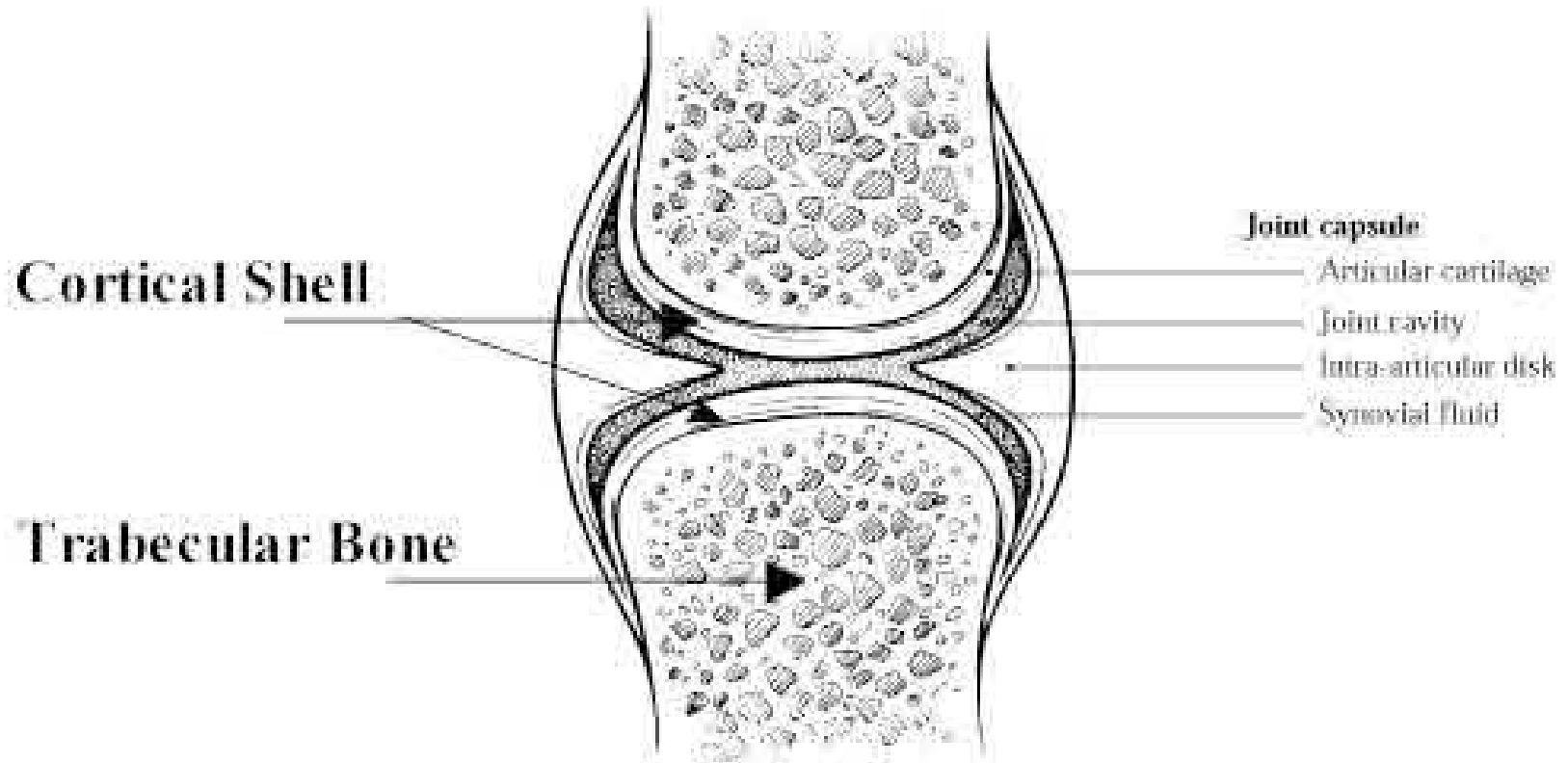
Osteoporosis is a bone disease in which the amount of bone is decreased and the structural integrity of trabecular bone is impaired. Cortical bone becomes more porous and thinner. This makes the bone weaker and more likely to fracture.



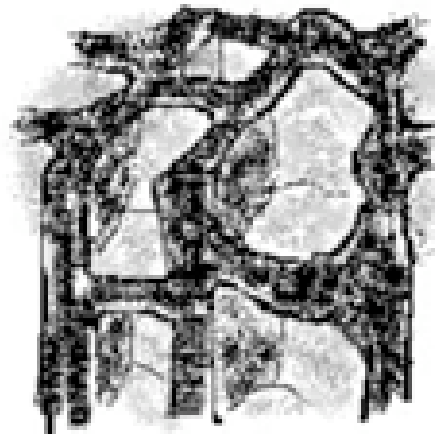
Loss of mineral
and microstructure



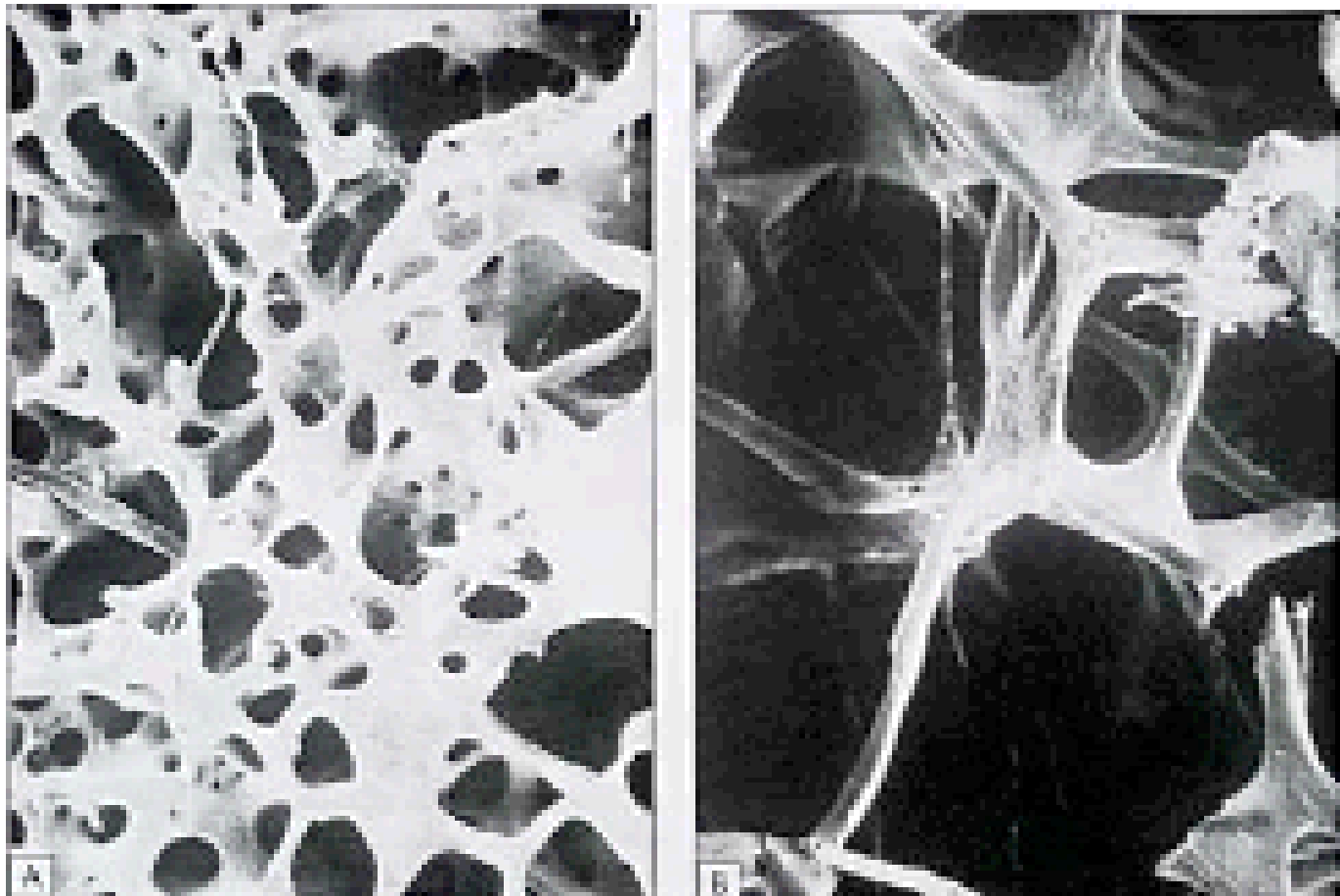
Cortical and Trabecular Bone



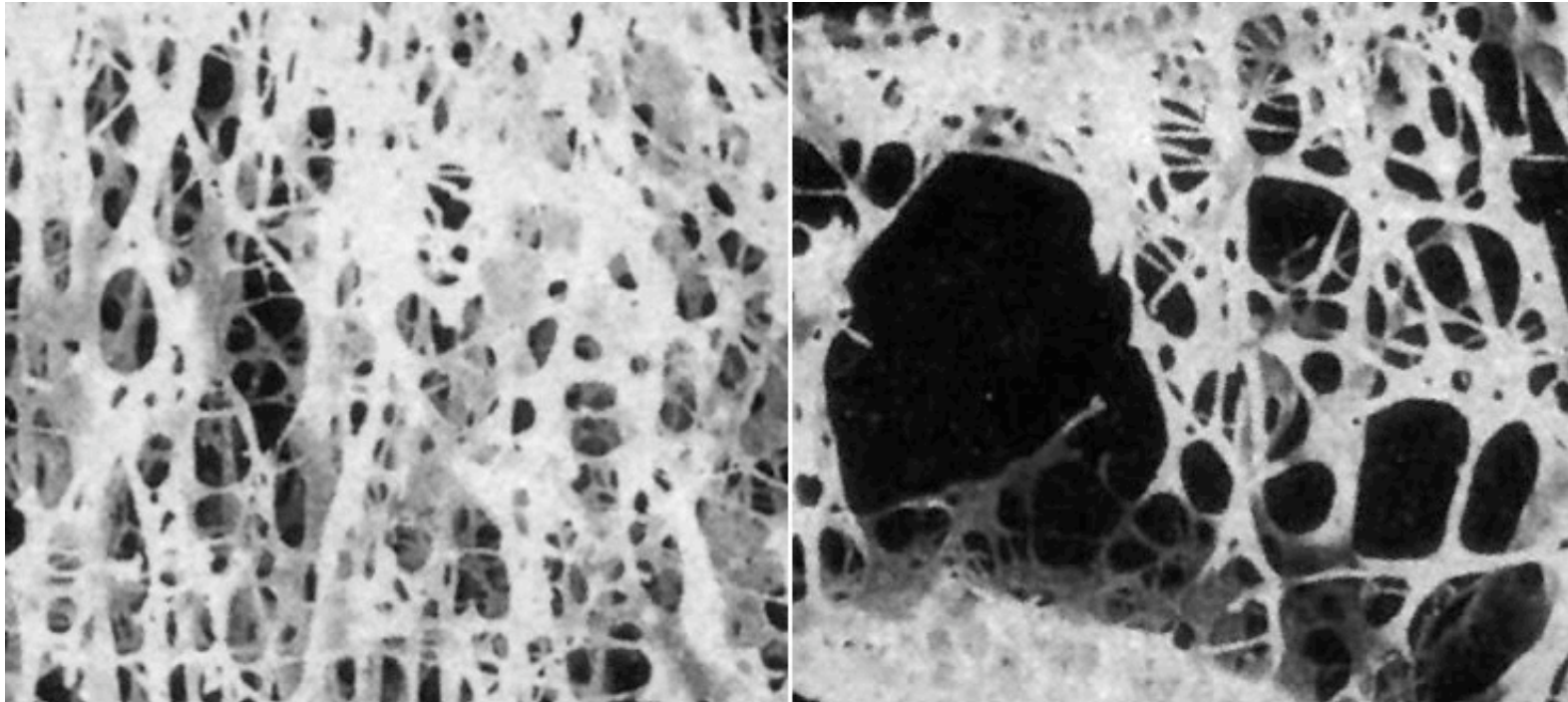
Healthy v. Osteoporotic Bone



Healthy v. Osteoporotic Bone

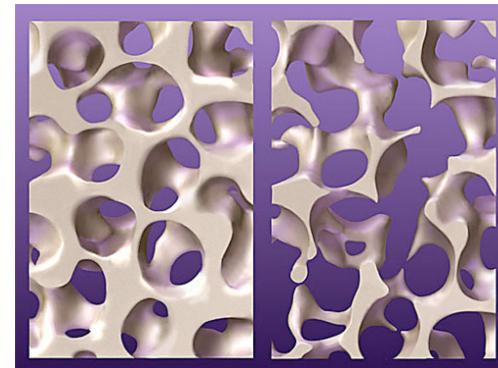
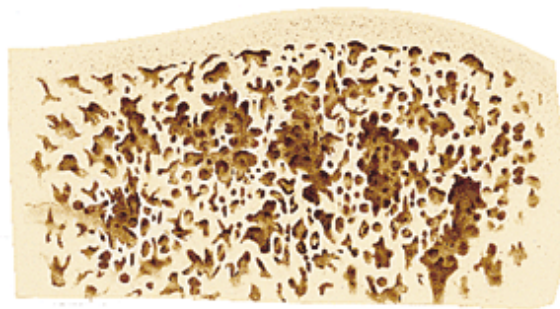


Healthy v. Osteoporotic Bone



Random Heterogeneous Material

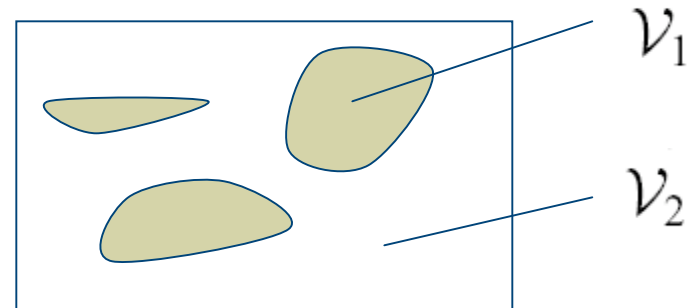
- ◆ Heterogeneous material refers to materials with more than one component, or phases.
- ◆ Here we have one phase as void and the other phase as bone, hence it's a two-phase material.
- ◆ We use the term random under the assumption that any sample of this medium is a realization of a specific random or stochastic process.



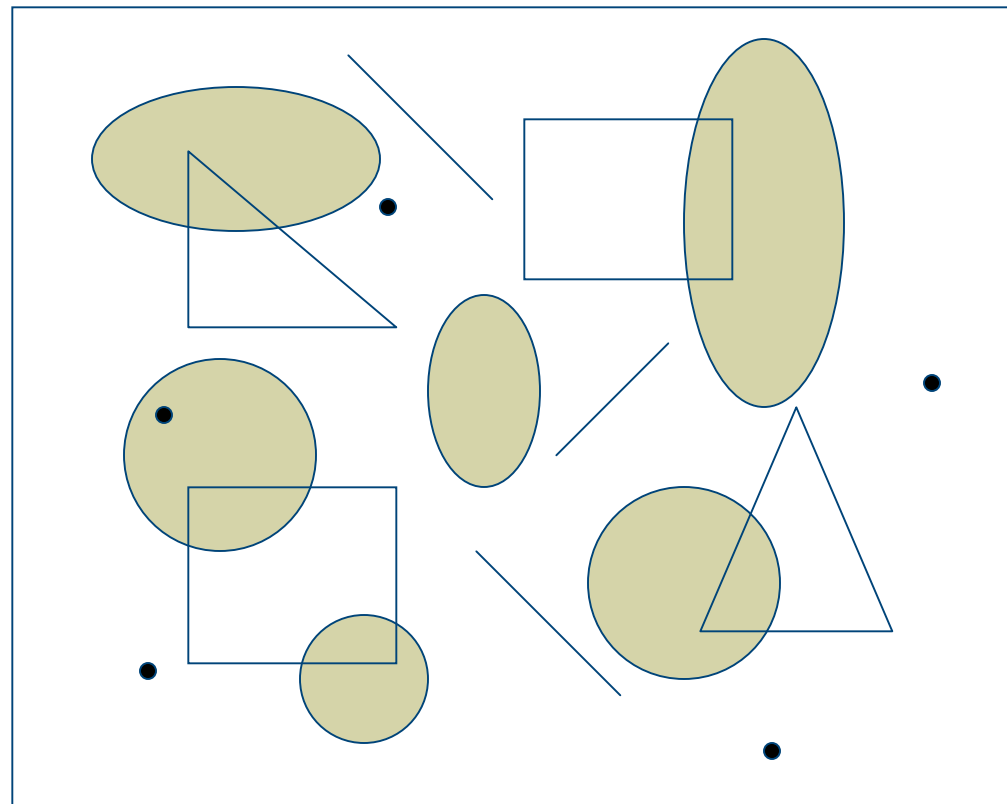
Volume and the Structure Function

- ◆ We assume the structure function is independent of time so that each realization of the medium is a region of the volume that is divided into phases: phase 1, \mathcal{V}_1 of volume fraction ϕ_1 ; and phase 2, \mathcal{V}_2 of volume fraction ϕ_2 .
- ◆ The sets are disjoint, meaning they have no intersection.
- ◆ The sum of the phase volumes must equal the material volume.
- ◆ For any given realization, the structure function is just the indicator function.

$$I(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \mathcal{V}_1 \\ 0 & \text{if } \mathbf{x} \in \mathcal{V}_2. \end{cases}$$



Correlation Functions



One-point Correlation Function

- ◆ We express a one point correlation function as the ensemble average over all realizations.
- ◆ When we average these we get the volume fraction of the component. Thus $\phi_1 = \langle I(\mathbf{x}) \rangle$.
- ◆ The one-point correlation function tells us the porosity of the bone. More porosity implies more void space. This generally means the bone is osteoporotic or osteopeniac.

Two-point Correlation Function

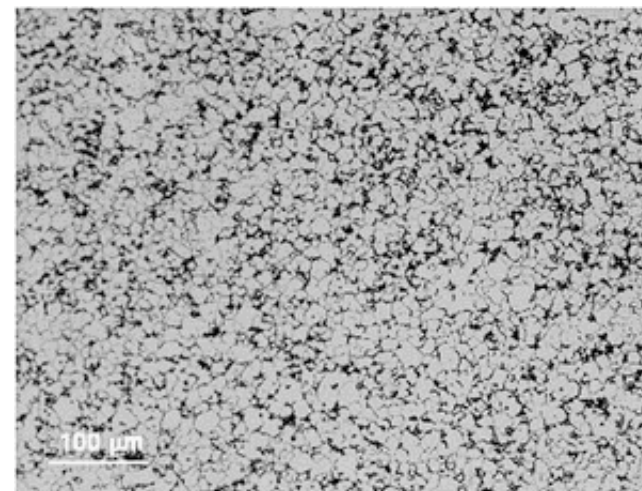
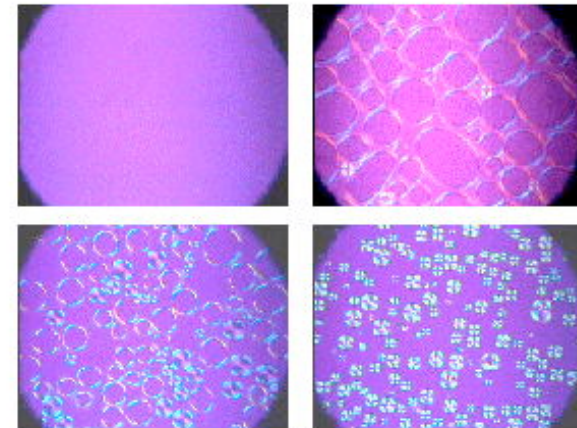
- ◆ The two-point correlation function is

$$S_2(\mathbf{x}_1, \mathbf{x}_2) = \langle I(\mathbf{x}_1)I(\mathbf{x}_2) \rangle$$

- ◆ Recall that this represents the probability that two points, \mathbf{x}_1 and \mathbf{x}_2 , both lie in phase 1.
- ◆ I let phase 1 represent the void portion of the bone. Then the function $S_2(\mathbf{x}_1, \mathbf{x}_2)$ represents the probability that two points both lie in a void space.

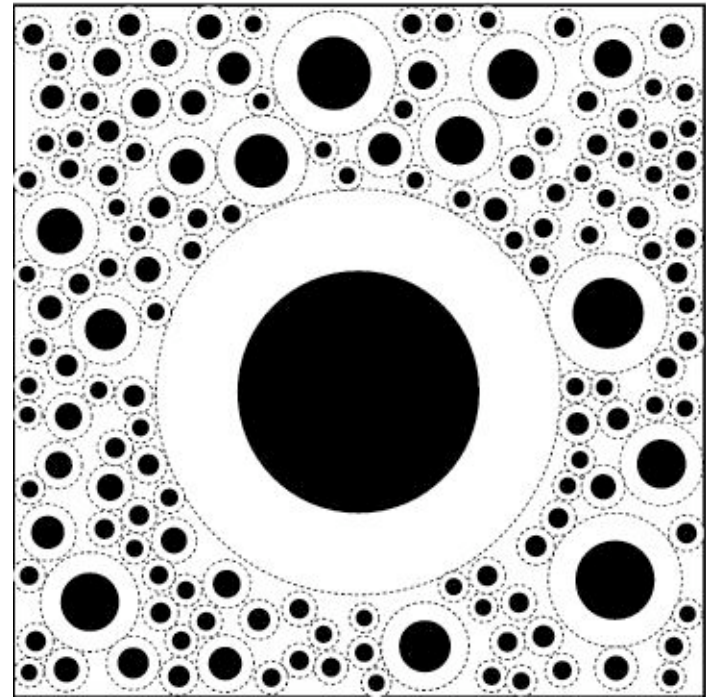
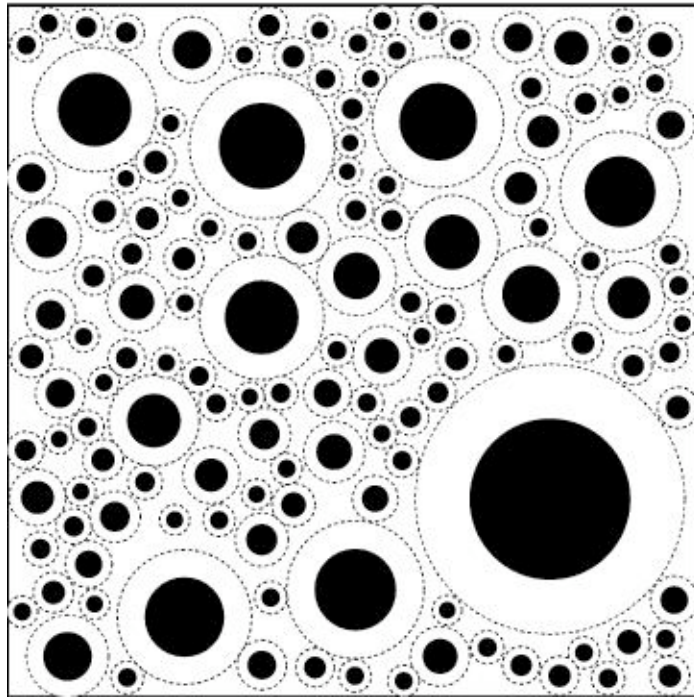
Two-point Correlation Function

- ◆ If the material is isotropic, the function depends only on the length of the segment, $r = |\mathbf{x}_1 - \mathbf{x}_2|$. We write this as $S_2(r)$.
- ◆ If the material is homogeneous, then we take an average of the volume, which is a much easier calculation.



Pictures generated by Nelson Beebe's program

The program generates Hashin-Shtrikman coated spheres





The Plan

- ◆ Create images using the program.
- ◆ Convert eps images into jpg images.
- ◆ “Read” the images using Matlab.
- ◆ Normalize the color scale to 0 and 1, where 0 represents white and 1 represents black (void).



What would all of this get us?..

- ◆ I might have used this information to estimate homogenized or effective properties of the bone structure.
- ◆ Homogenizing an elastic modulus allows us to predict the reaction of the material to stress and strain.
- ◆ Homogenizing bone structures would allow us to predict the reaction of the bone to stress and how likely would be the risk of fracture.
- ◆ It would also be helpful, and interesting, to be able to compare the two-point correlation functions of both healthy and diseased bone.