

## Math 6510 - Homework 1

Due in class on 9/12/13

1. Find a differentiable atlas for  $S^1 \times S^1$ ,
2. Show that if  $M$  and  $N$  are differentiable manifolds then  $M \times N$  is a differentiable manifold.
3. Find a differentiable atlas of  $\mathbb{R}$  such that the identity map is not smooth.
4. Let  $U, V \subset \mathbb{R}^n$  be open and  $f : U \rightarrow V$  a smooth homeomorphism such that  $f_*(x)$  is an isomorphism for some  $x \in U$ . Show that  $f^{-1}$  is differentiable at  $y = f(x)$  and that  $(f^{-1})_*(y) = (f_*(x))^{-1}$ .
5. For  $i = 0, 1$  let  $U_i \subset \mathbb{R}^k$  be open and  $\phi_i : U_i \rightarrow \mathbb{R}^n$  smooth, injective maps such that  $\phi_0(U_0) = \phi_1(U_1)$  and  $(\phi_i)_*(x)$  is injective for all  $i \in U_i$ . Show that there exists a diffeomorphism  $f : U_0 \rightarrow U_1$  such that  $\phi_0 = \phi_1 \circ f$ .