Math 6510 - Homework 5

Due in class on 11/18/14

1. Let M_0, M_1 and N be differentiable manifolds and $f : M_0 \times M_1 \to N$ a smooth map. Then the map $F : M_0 \times TM_1 \to TN$ defined by $F(x_0, v) = (f_*(x_0, x_1))(0, v)$ is smooth where $v \in T_{x_1}M_1$. (You don't need to prove this but you should make sure that you know why its true!).

   Let G be a Lie group and define $f : G \times G \to G$ by $f(a, b) = ab$. Then use the above fact to show that a left-invariant vector field is smooth by showing that F restricted to $G \times \{v\}$ is an embedding where $v \in T_{id}G$.

2. Let G be a Lie group and $\mathfrak{g}$ its Lie algebra. Define a vector field V on $G \times \mathfrak{g}$ by $V(g, v) = ((L_g)_*(id)v, 0)$. Show that V is smooth and that it has a flow $\Phi : G \times \mathfrak{g} \times \mathbb{R} \to G \times \mathfrak{g}$ defined for all time. Define a map $\exp : \mathfrak{g} \to G$ by $\pi \circ \Phi(id, v, 1)$ where $\pi : G \times \mathfrak{g} \to G$ is the projection onto the first factor. Show that $\exp_*(0) = id$.