

Name:

**Midterm 2, Math 3210**  
**October 23rd, 2015**

**You must write in complete sentences and justify all of your work. Do 3 of the 4 problems below. All 3 problems that you do will be equally weighted.**

**Clearly mark in the table below which 3 problems you want graded.**

Problem	1	2	3	4
Graded?				
Score				

1. Directly using the definition of a limit show that  $\lim_{n \rightarrow \infty} \sqrt{n^2 + 1} - n = 0$ .

2. Let  $a_n$  be a sequence of positive numbers and assume that the sequence  $b_n = a_n/n$  converges to some  $b > 0$ . Show that there exists a constant  $c > 0$  such that  $a_n \geq cn$  for all positive integers  $n$ .

3. Directly using the definition of a Cauchy sequence show that  $a_n = \frac{1}{2n}$  is a Cauchy sequence.

4. Let  $f : [0, 1] \rightarrow [-1, 0]$  be a continuous function. Show that there exists an  $x \in [0, 1]$  such that  $f(x) = -x$ .

## Scratchwork