

Name:

Midterm 3, Math 3210
November 20th, 2015

You must write in complete sentences and justify all of your work. Do 3 of the 4 problems below. All 3 problems that you do will be equally weighted. Clearly mark in the table below which 3 problems you want graded.

Problem	1	2	3	4
Graded?				
Score				

1. Let $f: (a, b) \rightarrow \mathbb{R}$ be uniformly continuous and assume that $g: (a, b) \rightarrow \mathbb{R}$ is another function such that there exists a $K > 0$ with

$$|g(x) - g(y)| \leq K|f(x) - f(y)|$$

for all $x, y \in (a, b)$. Show that g is uniformly continuous.

2. Define functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

and

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

For both functions either find the derivative at 0 (with proof) or show that it doesn't exist.

3. Let $f: [0, 1] \rightarrow \mathbb{R}$ be a continuous function that is differentiable on $(0, 1)$. Assume that $f(0) = 1$ and $f'(x) > -1$ for all $x \in (0, 1)$. Show that $f(x) > 0$ for all $x \in [0, 1]$.

4. Define $f: [0, 1] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & \text{if } x < 1/2 \\ 1 & \text{if } x \geq 1/2 \end{cases}$$

and define partitions $P_n = \{0 < \frac{1}{2} - \frac{1}{n} < \frac{1}{2} + \frac{1}{n} < 1\}$. Calculate $U(f, P_n)$ and $L(f, P_n)$. Is f integrable? Make sure to justify your answer.

Scratchwork