Final, Math 6250 Algebraic Topology
Spring 2008

Instructions: Do four (4) problems in 1.5 hours. Be sure to provide all relevant definitions and statements of theorems cited. Make sure you indicate which solutions are to be graded, otherwise the first four answered will be scored.

A. Answer four of the following questions. Each question is worth ten points.

1. Let M be an open 3-manifold such that every compact set of M is contained in a solid torus.
   (a) Show that $\pi_1(M)$ is abelian.
   (b) Assume that $\pi_1(M)$ is finitely generated. Show that $\pi_1(M)$ is trivial or $\mathbb{Z}$.
2. Let $f : S^n \longrightarrow S^n$ be a continuous map from the n-sphere to itself and assume that n is even. Show that there is a point $x \in S^n$ such that $f(x) \in \{-x, x\}$. Does the same assertion hold if n is odd? If it does, prove it, if it does not construct a counterexample.
3. Let c be a non-separating simple closed curve on a compact surface S. Show that S doesn't retract onto c.
4. Construct a Δ-complex for the Klein bottle and calculate its simplicial homology for both $\mathbb{Z}$ and $\mathbb{Z}_2$ coefficients.
5. Let M be a Möbius strip and D a disk. Let $f : \partial D \longrightarrow \partial M$ have degree n. Set $X_n = M \amalg D/\sim$ where $x \sim y$ if $f(x) = y$. Calculate $\pi_1(X_k)$ and construct all of its connected covering spaces.
6. Construct a homeomorphism $f : S^1 \times S^1 \longrightarrow S^1 \times S^1$ that is not homotopic to a map without fixed points.
7. Show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$.
8. Let X_k be the space obtained by taking repeated suspensions of a space X; i.e. X_k is the suspension of X_{k-1}. Assume that for

some k, X_k is an orientable n-dimensional manifold. Calculate the homology of X.