Midterm 1, Math 3210
February 4, 2013
You must write in complete sentences and justify all of your work. All 3 problems will be equally weighted.

1. Use induction to prove that
$$\sum_{k=1}^{n}(2k-1) = n^2$$
for all $n \in \mathbb{N}$.

2. Recall that a subset L of the rationals is a Dedekind cut if:
   (a) $L \neq \emptyset$ and $L \neq \mathbb{Q}$;
   (b) L has no largest element;
   (c) If $x \in L$ and $y \in \mathbb{Q}$ with $y < x$ then $y \in L$.

   If L is a Dedekind cut show that the set
$$K = \{x \in \mathbb{Q} | \exists y \in L \text{ with } x + y < 0\}$$
   is a Dedekind cut.

3. Let $a_{n-1}, \ldots, a_0$ be integers. Show that if r is a rational number with
$$2r^n + a_{n_1} r^{n-1} + \cdots + a_0 = 0$$
   then $2r$ is an integer.