Name:

Midterm 1, Math 3210
February 2, 2018
You must write in complete sentences and justify all of your work. All 3 problems will be equally weighted.

Problem	**1**	**2**	**3**	**Total**
Score				

1. Use induction to prove that

$$2^n < 3^n$$

for all $n \in \mathbb{N}$. You can use all of the usual properties of addition, multiplication and order for the natural numbers (but not properties of exponents).

2. For the following you should assume that x, y and z are elements of a field F as defined in the book and notes.

   (a) Prove that if $xz = yz$ and $z \neq 0$ then $x = y$.

   (b) Prove that $xy = 0$ then either $x = 0$ or $y = 0$.

   In your proofs you can only use the properties of a field given in the notes along with the following two results we proved in class:

   (i) If $x + z = y + z$ then $z = y$.

   (ii) $x \cdot 0 = 0$.

3. If L is a Dedekind cut show that the set

$$K = \{x \in \mathbb{Q} | \exists y \in L \text{ with } x = y + 1\}$$

is a Dedekind cut.

Peano Axioms. The set of natural numbers $\mathbb{N}$ satisfy the following properties:

N1. There is an element $1 \in \mathbb{N}$.

N2. There is a *successor* function $s : \mathbb{N} \to \mathbb{N}$.

N3. $1 \notin s(\mathbb{N})$.

N4. The successor function s is injective. That is if $s(n) = s(m)$ then $n = m$.

N5. If $A \subset \mathbb{N}$ contains 1 and $s(A) \subset A$ then $A = \mathbb{N}$.

Rings and fields. A field F is a set with operations of additions and multiplication that satisfy:

A1. $x + y = y + x$ for all $x, y \in F$.

A2. $(x + y) + z = x + (y + z)$ for all $x, y, z \in F$.

A3. There exists a $0 \in F$ such that $x + 0 = x$ for all $x \in F$.

A4. For all $x \in F$ there exists and $-x \in F$ such that $x + (-x) = 0$.

M1. $xy = yx$ for all $x, y \in F$.

M2. $(xy)z = x(yz)$ for all $x, y, z \in F$.

M3. There exists a $1 \in F$ with $1 \neq 0$ such that $x \cdot 1 = x$ for all $x \in F$.

M4. For all $x \in F$ with $x \neq 0$, there exists an $x^{-1} \in F$ with $xx^{-1} = 1$.

D. $x(y + z) = xy + xz$ for all $x, y, z \in F$.

Dedekind cuts. A set $L \subset \mathbb{Q}$ is a Dedekind cut if

(a) $L \neq \emptyset, \mathbb{Q}$.

(b) There does not exists a $r \in L$ such that $r \geq x$ for all $x \in L$. (L has no largest element.)

(c) If $x \in L$ then for all $y \in \mathbb{Q}$ with $y < x$, $y \in L$.

Scratch Work