Name:

Midterm 3, Math 3210
April 6, 2018

You must write in complete sentences and justify all of your work. All 4 problems will be equally weighted.

Problem	1	2	3	4	Total
Score					

1. Show that the following functions are uniformly continuous on the given domain or prove that they are not:

   (a) $f(x) = x^2$ with domain $[0, 1]$.

   (b) $g(x) = 1/x$ with domain $(0, 1]$.

2. Let $f\colon (-1,1) \to \mathbb{R}$ be a continuous (but not necessarily differentiable) function with $f(0) = 1$ and let $g(x) = xf(x)$. Show that $g'(0) = 1$. (In particular show that $g'(0)$ exists.)

3. Let $f(x) = \sqrt{x+1}$ and show that $f'(x) \leq \frac{1}{2}$ if $x \geq 0$. Use this and the Mean Value Theorem to show that $\sqrt{x+1} \leq 1 + \frac{x}{2}$ if $x \geq 0$.

4. Let $f(x) = 1/x$ with domain $[1, 2]$.

(a) Let $P = \{1 < 3/2 < 2\}$ be a partition and calculate the upper and lower sums $U(f, P)$ and $L(f, P)$.

(b) Use the previous problem to give upper and lower bounds for the integral

$$\int_1^2 f(x)dx.$$

Scratchwork