MATH 2270

Exam #1 - Fall 2008

Name: ______________________________

1. (15 points) Let $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \\ 1 & 3 & 7 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$.

   (a) Solve the linear system $A\vec{x} = \vec{b}$ for $\vec{x}$ using Gauss-Jordan elimination. Clearly show each step and indicate your answer. If the system is inconsistent, write *inconsistent*.

(b) what is rref(A) (i.e. the reduced row echelon form of A)?

(c) what is rank(A)? Why?

(d) Is the matrix A invertible? Why?

(e) Find a basis for the kernel of A (i.e. ker(A)).

(f) Find a basis for the image of A (i.e. im(A)).

2. (8 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}.$$

(a) Prove T is a linear transformation.

(b) Write the matrix corresponding to T.

(c) Give an example of a vector in $\mathbb{R}^3$ not contained in either $\operatorname{im}(T)$ or $\ker(T)$.

3. (4 points) Let $k \in \mathbb{R}$. Compute the inverse of the following matrix using Gauss-Jordan elimination. Show your work.

$$A = \begin{pmatrix} 1 & k \\ 0 & 2 \end{pmatrix}.$$

4. (2 points) Let $a, b, c, d \in \mathbb{R}$. Compute the following matrix products.

(a) $\begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} c & d \end{pmatrix} =$

(b) $\begin{pmatrix} c & d \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} =$

5. (4 points) Consider the $n \times 4$ matrix

$$A = \begin{pmatrix} | & | & | & | \\ \vec{v_1} & \vec{v_2} & \vec{v_3} & \vec{v_4} \\ | & | & | & | \end{pmatrix}.$$

You are told the vector $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ is an element of $\ker(A)$. Write $\vec{v_4}$ as a linear combination of the vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$.

6. (4 points) Multiple choice — choose the best answer for the following questions.

   (a) Suppose two distinct solutions, $\vec{x_1}$ and $\vec{x_2}$, can be found for the linear system $A\vec{x} = \vec{b}$. Which of the following is necessarily true?

      i. $\vec{b} = \vec{0}$.
      ii. A is invertible.
      iii. A has more columns that rows.
      iv. $\vec{x_1} = -\vec{x_2}$
      v. There exists a solution $\vec{x}$ such that $\vec{x} \neq \vec{x_1}$ and $\vec{x} \neq \vec{x_2}$.

   (b) Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation whose kernel is a 3-dimensional subspace of $\mathbb{R}^5$. Then im(T) is

      i. The trivial subspace.
      ii. A line through the origin.
      iii. A plane through the origin.
      iv. all of $\mathbb{R}^3$.
      v. Cannot be determined from the given information.

7. (3 points) Compute the following matrix product.

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^{100} - \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{100}.$$