MATH 2270

Exam #3 - Fall 2008

Name: ____________________

1. (4 points) Find the determinant of the following matrix. Show your work.

$$\begin{pmatrix} 0 & 3 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

2. (6 points) Consider a 3×3 matrix A with rows $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$

$$A = \begin{pmatrix} - & \vec{v}_1 & - \\ - & \vec{v}_2 & - \\ - & \vec{v}_3 & - \end{pmatrix}$$

and suppose $\det(A) = 8$.

(a) Find $\det \begin{pmatrix} 2\vec{v}_1 \\ \vec{v}_3 \\ \vec{v}_2 \end{pmatrix}$.

(b) Find $\det(A^{-1}A^T A)$.

(c) Suppose the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ are orthogonal and have norm x, i.e.

$$\|\vec{v}_1\| = \|\vec{v}_2\| = \|\vec{v}_3\| = x.$$

Find x.

3. (8 points) For $\alpha \in \mathbb{R}$, let

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & \alpha \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

and observe that the only eigenvalues of A are $\lambda = 1$ and $\lambda = 2$.

(a) What is the algebraic multiplicity of the eigenvalue $\lambda = 1$?

(b) What is the geometric multiplicity of the eigenvalue $\lambda = 1$?

(c) For which values of α will the geometric multiplicity of $\lambda = 2$ and the algebraic multiplicity of $\lambda = 2$ be equal?

(d) If $\alpha = 0$, is A diagonalizable? Explain your answer.

4. (8 points) Suppose

$$A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}.$$

Find a diagonal matrix D and an orthogonal matrix S such that $D = S^T AS$. Show your work.

5. (10 points)

(a) Is the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ similar to the matrix $\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$?

(b) Is the matrix $\begin{pmatrix} -1 & 6 \\ -2 & 6 \end{pmatrix}$ similar to the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$?

(c) Suppose $\vec{v}$ is an eigenvector of an invertible $n \times n$ matrix A and let λ be the associated eigenvalue. Is $\vec{v}$ an eigenvector of $A^{-1} + 2I_n$? If so, what is the associated eigenvalue? If not, explain why.

(d) Let $q(x_1, x_2, x_3) = 3x_1^2 + 4x_2^2 + 5x_3^2 - 6x_1x_2 + 7x_2x_3$ be a quadratic form. Find a symmetric 3×3 matrix A such that $q(\vec{x}) = \vec{x}^T A \vec{x}$.

(e) Suppose A is a symmetric, orthogonal $n \times n$ matrix. Find the eigenvalues (with multiplicities) of A^2.

6. (4 points) Multiple choice. Choose the best answer to each of the following questions.

(a) If a 2×2 matrix A has eigenvalues $\lambda = 1$ and $\lambda = 2$, then trace(A^{100}) is

i. 2^{100}

ii. 2^{101}

iii. $2^{100} + 1$

iv. 3^{100}

v. Cannot be determined from the given information.

(b) If A is a symmetric positive definite 2×2 matrix, which of the following must be true?

i. The algebraic multiplicity of every eigenvalue is 1.

ii. A^{-1} is negative definite.

iii. $\det(-A) < 0$

iv. The curve defined by the equation $\vec{x}^T A \vec{x} = 1$ is an ellipse.

v. none of the above.