MATH 2270

Quiz #4 - Fall 2008

Name: Answer

1. (5 points) Find the matrix B of the linear transformation

$$T(\vec{x}) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \vec{x}$$

with respect to the basis

$$\mathfrak{B} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}.$$

Method 1:

Let $S = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ so that $S^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$. Then

$$B = S^{-1} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} S = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 7 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} -2 & -4 \\ 3 & 7 \end{pmatrix}}$$

Method 2:

$$B = \begin{pmatrix} | & | \\ \left(T\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)_{\mathfrak{B}} & \left(T\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)_{\mathfrak{B}} \\ | & | \end{pmatrix}. \quad \text{So}$$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}_{\mathfrak{B}}.$$

$$T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}_{\mathfrak{B}}.$$

$$\therefore \boxed{B = \begin{pmatrix} -2 & -4 \\ 3 & 7 \end{pmatrix}}$$

2. (4 points) Prove the set of 3×3 matrices A such that the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is in the kernel of A is a subspace of $\mathbb{R}^{3\times 3}$.

- Clearly 0 is in this set since every vector is in ~~ker~~ ker(0)

- If A, B are two such matrices, then

$$(A+B)\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + B\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \vec{0} + \vec{0} = \vec{0}.$$

So the set is closed under <u>addition</u>

- If A is such a matrix, the

$$(\alpha A)\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \alpha\left(A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\right) = \alpha \cdot \vec{0} = \vec{0},$$

So the set is closed under scalar multiplicatn

3. (2 points) True or false. Indicate whether the following statements are true or false.

(a) The function $T(M) = 7M$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$ is a linear transformation.

True

(b) The function $T(M) = M^2$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$ is a linear transformation.

False