MATH 2270

Quiz #8 - Fall 2008

Name: Answer Key

1. (5 points) Find an eigenbasis for the matrix

$$A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}.$$

$$f_A(\lambda) = \lambda^2 - \lambda = \lambda(\lambda - 1).$$

eigenvalues: $0, 1$

$$E_0 = \ker \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = \operatorname{span}\left\{ \begin{pmatrix} +1 \\ +1 \end{pmatrix} \right\}$$

$$E_1 = \ker \begin{pmatrix} 1 & -2 \\ 1 & -2 \end{pmatrix} = \operatorname{span}\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

Therefore, an eigenbasis for A is given by

$$\mathcal{D} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

2. (4 points)

(a) For which values of a and b is the following matrix diagonalizable?

$$A = \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix}.$$

• If $b \neq 1$, then A has two distinct eigenvalues and is diagonalizable. If $b = 1$, then A is diagonalizable if and only if $a = 0$.

(b) For which values of a, b, and c is the following matrix diagonalizable?

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

• Since A is symmetric, A is diagonalizable by the spectral theorem. Therefore any values of a, b, c are allowed.

3. (2 points) True or false. Indicate whether the following statements are true or false.

(a) All invertible matrices are diagonalizable.

$\boxed{\text{False,}}$ for example $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(b) The algebraic multiplicity of an eigenvalue cannot exceed its geometric multiplicity.

$\boxed{\text{False,}}$ we always have

geometric multiplicty $\leq$ algebraic multiplicity