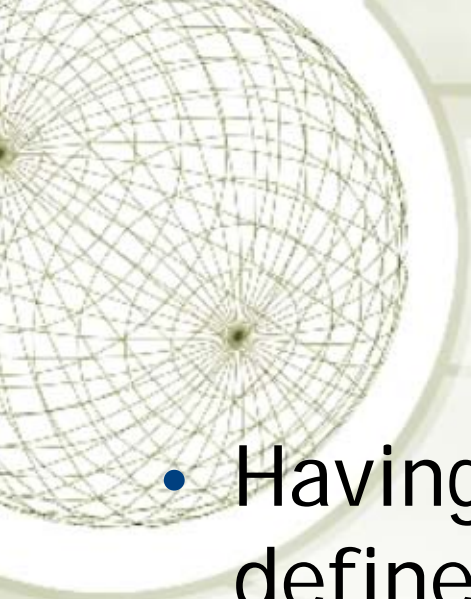




Geometries of 3-manifolds



Projective 3-space -- P^3

- Having the definition of P^2 in mind, define P^3 .
- What kind of geometry does it have?
- Is it orientable?

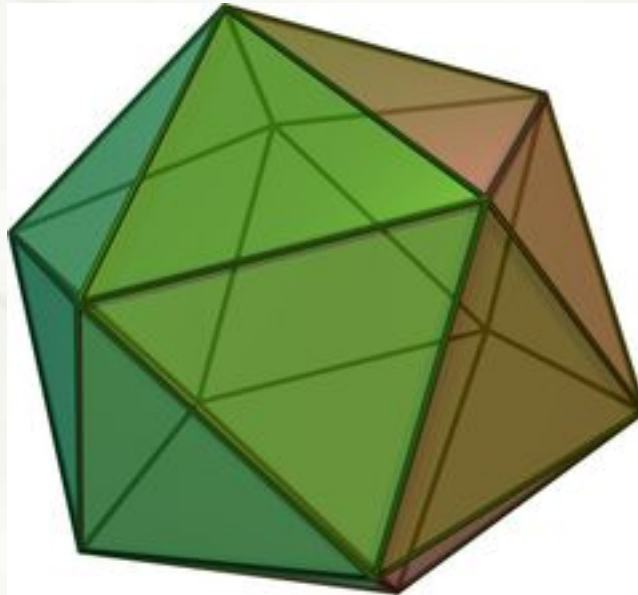
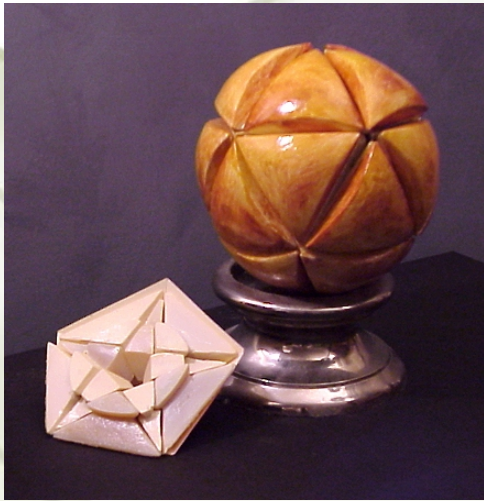


Hyperbolic 3-space

- Just “like” H^2 , but one dimension bigger.
- Every two dimensional slice is H^2 .

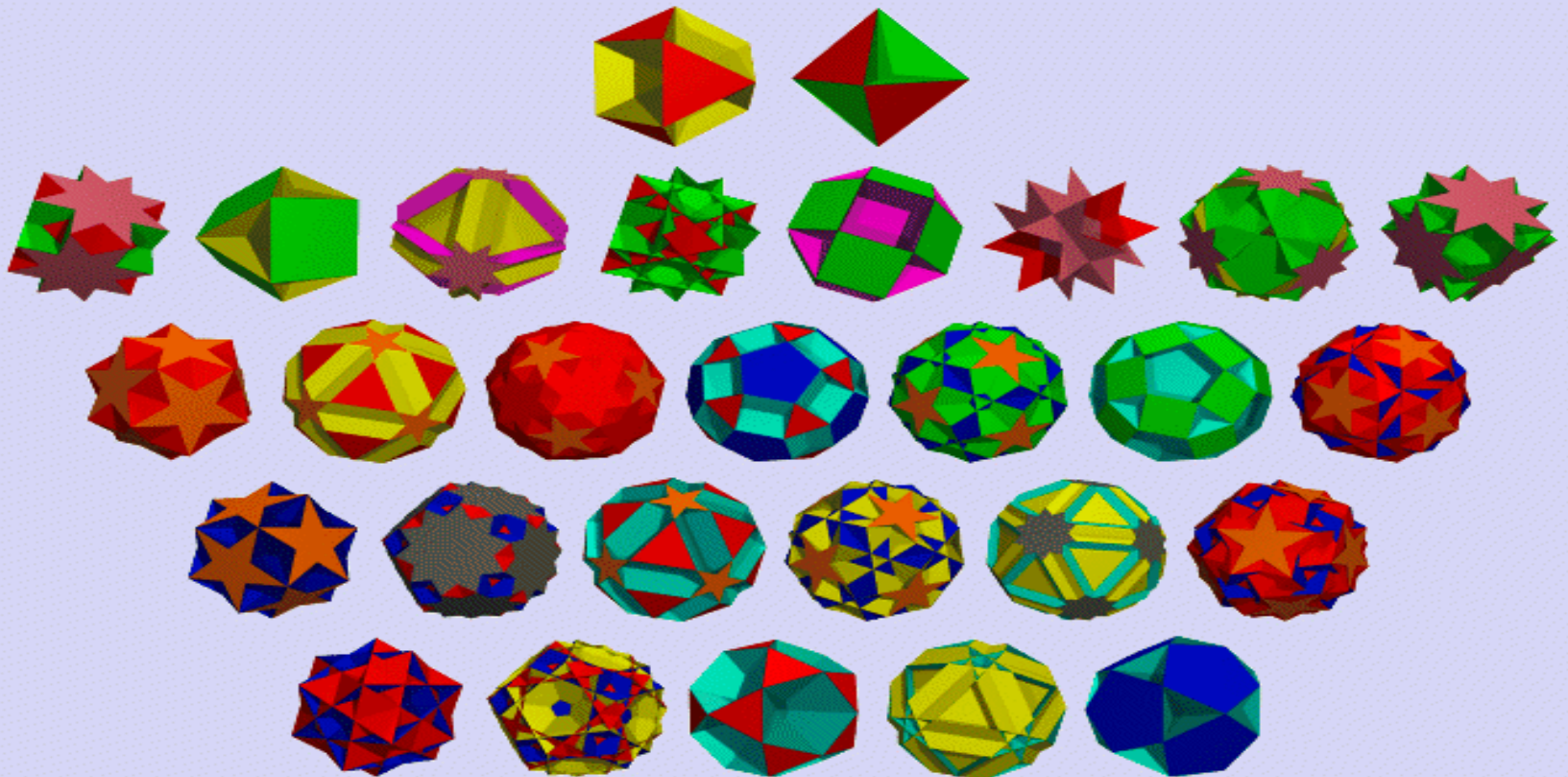
Differences?

- Polyhedra play the role of polygons



Polyhedron

- three - dimensional figure made up of sides called faces (polygons). In a polyhedron, several polygonal faces meet at a corner (vertex).





3-manifolds

- are likely to be made from polyhedra by identifying their sides
 - Gluing the sides of a cube in a certain way (which?) gives us a 3-torus.
- depending on how the corners of the polyhedron fit together we'll decide what type of geometry the 3-manifold has.

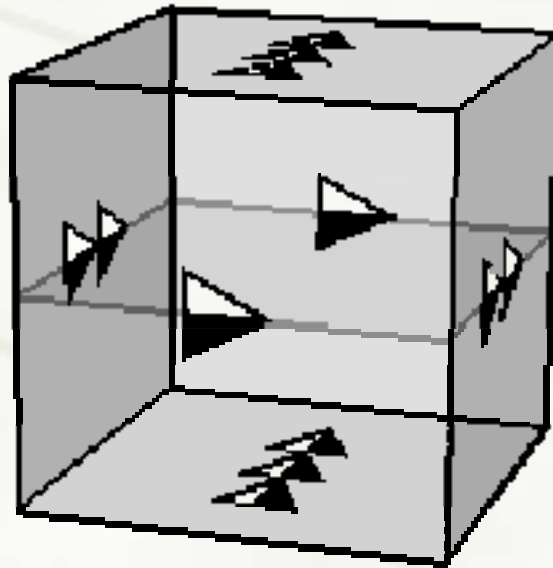


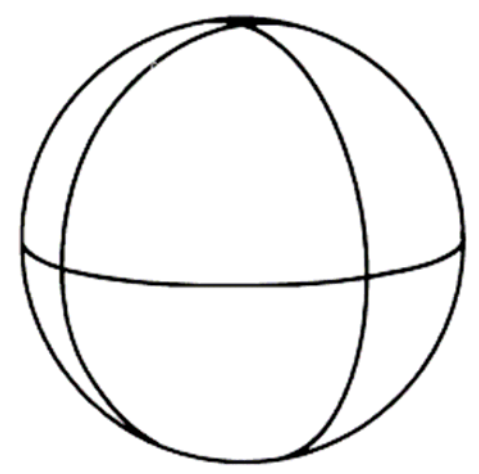
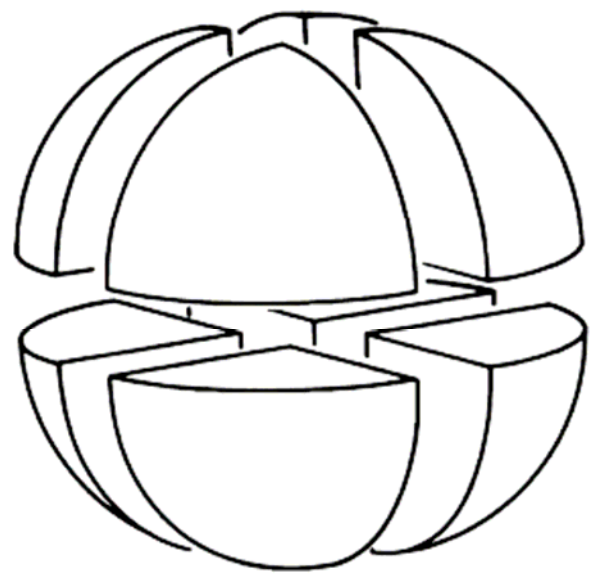
Comparison

- In a two manifold you wanted the corners of a polygon to fit into a circle around a vertex
- In a three manifold you'll want the corners of the polyhedron to fit into what shape around a vertex?

3-torus

- How do corners of a cube fit around a vertex in a 3-torus?

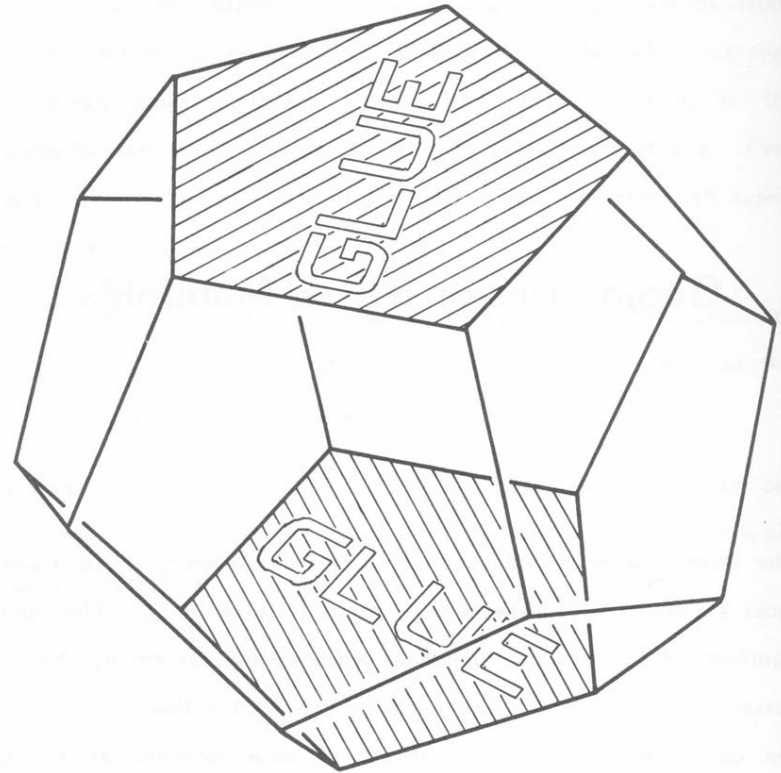




T^3 has flat geometry

Seifert-Weber space

Glue the opposite sides of the dodecahedron with a three-tenths of a clockwise turn.





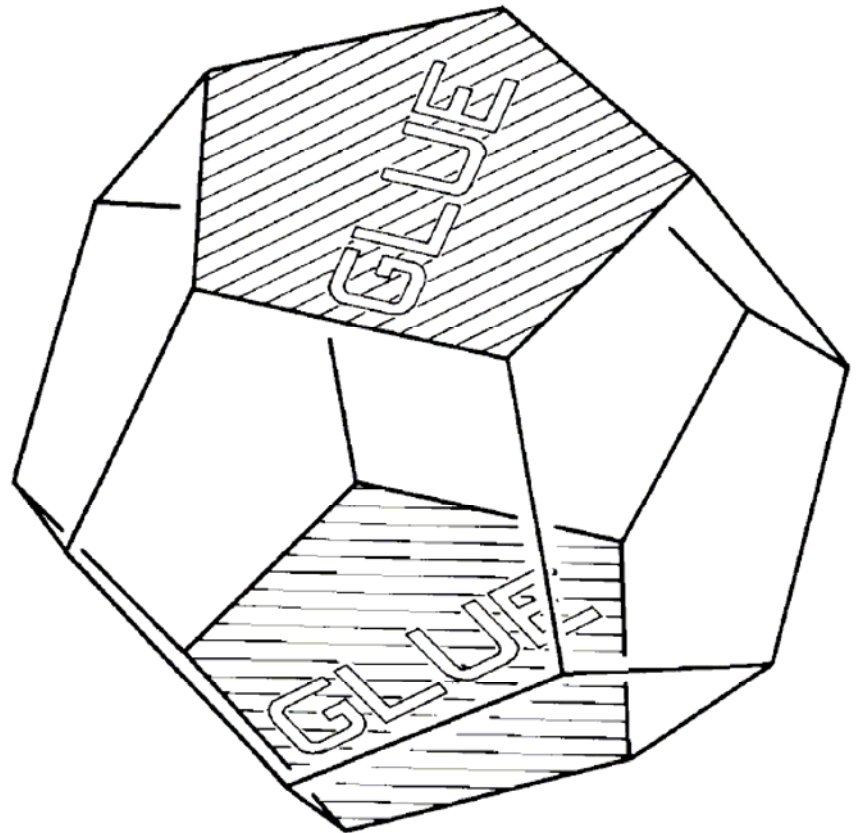
Geometry of S-W space

- Turns out that all the corners fit around the same point
- Too much space to surround one tiny point
- Put the dodecahedron into the H^3 and let it grow, until all the corners become small enough so that they can all fit around the vertex.

S-W space admits HYPERBOLIC GEOMETRY

Poincare dodecahedral space

The opposite faces are glued with a one tenth clockwise turn



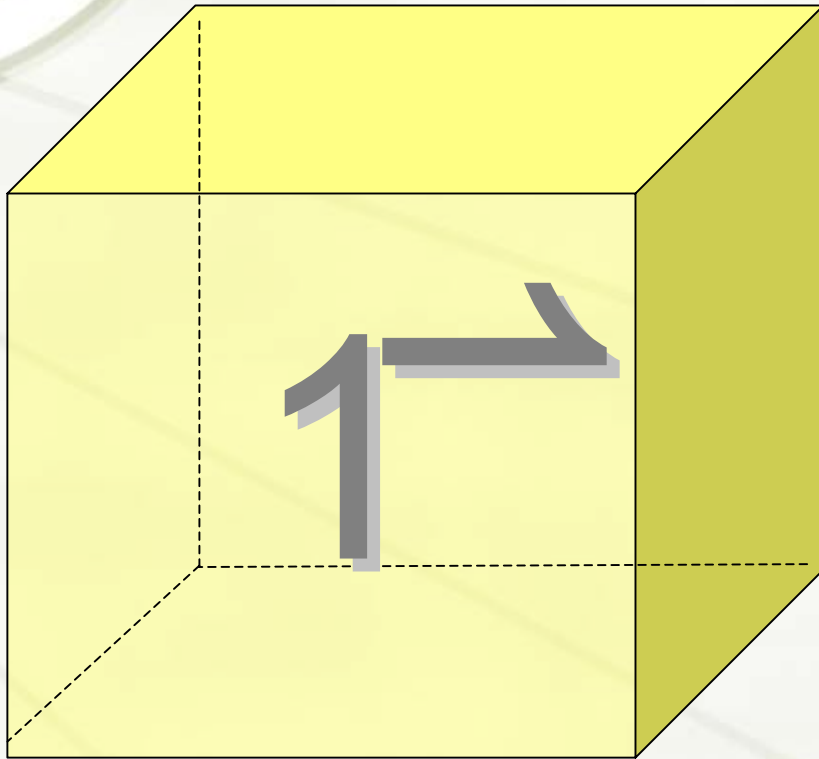


Geometry of Poincare space

- Turns out that there are 5 groups of 4 corners that come together
- Not enough space to surround one point
- Put the dodecahedron into the S^3 and let it grow, until all the corners become big enough.

Poincare space admits SPHERICAL GEOMETRY

Geometry?



Opposite sides are
glued with one
quarter clockwise
rotation



Is that it?

- Are those all geometries?
- Haven't we already seen one that's not either one of these three?
- Next time....