Connected Sums

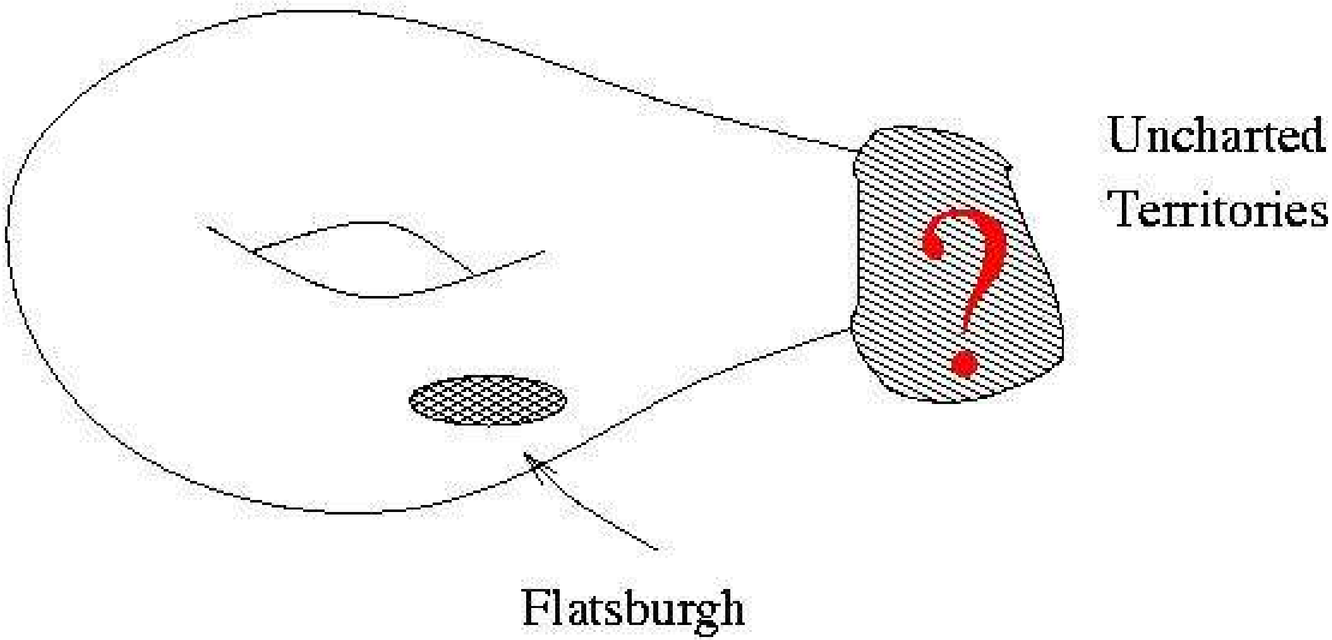
Exercise 1

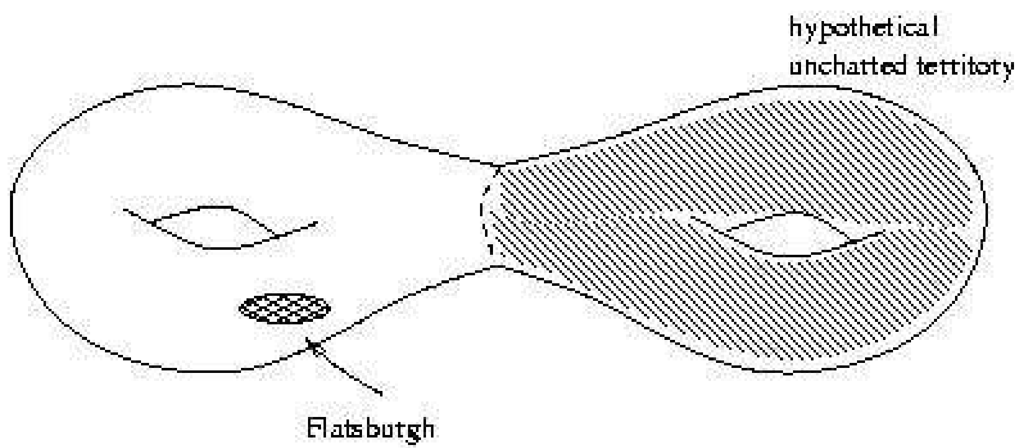
- Draw gluing diagrams of each surface we have discussed so far. State whether the manifold has flat or curved geometry and whether it is orientable or not.

Are these all surfaces that there are???

- Can we make more surfaces?
- If so, how would we do that?

We finish the story of Flatland

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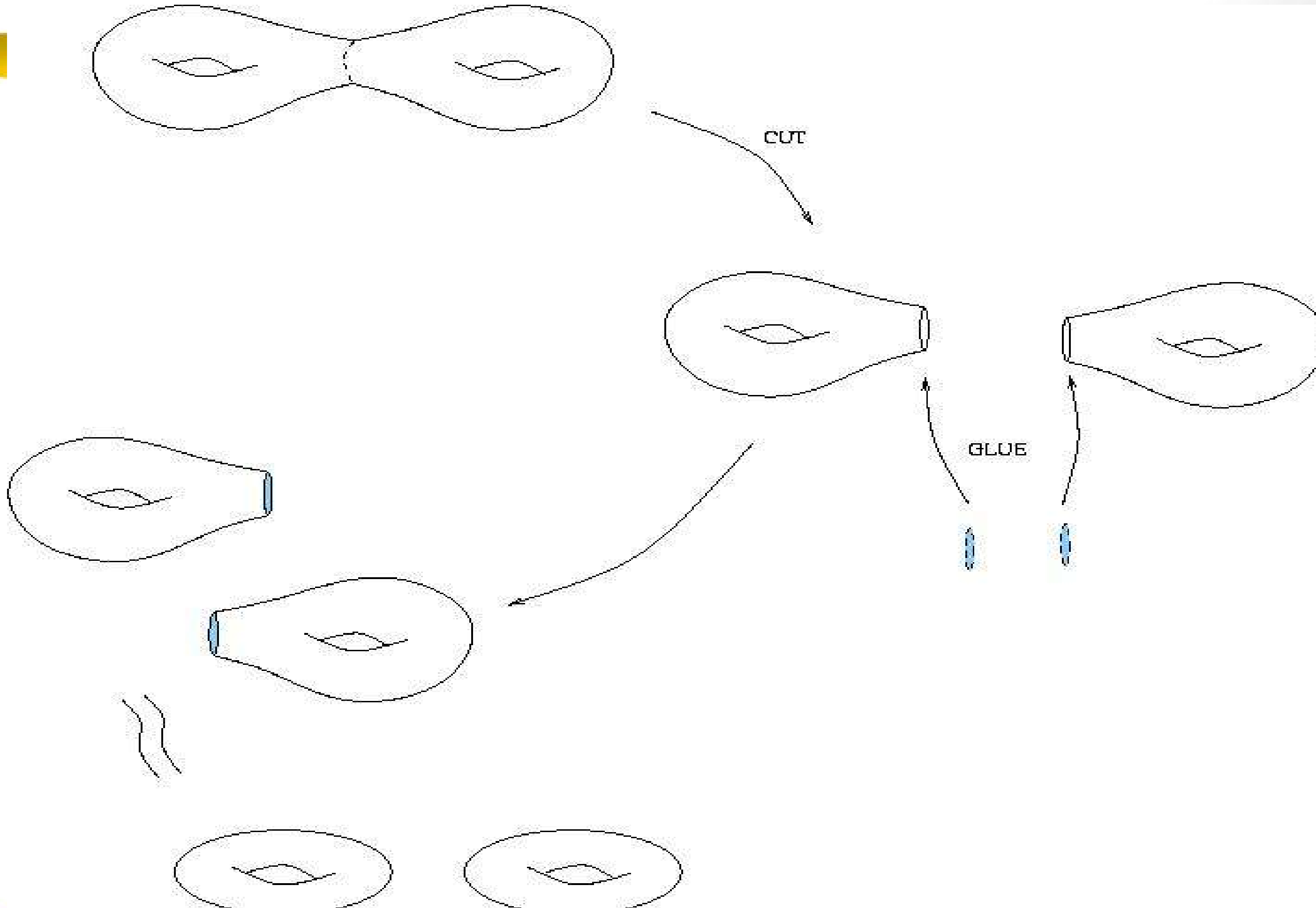
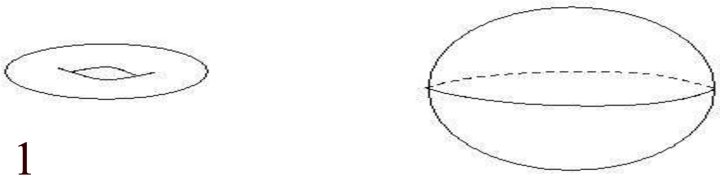
Break up



Getting together



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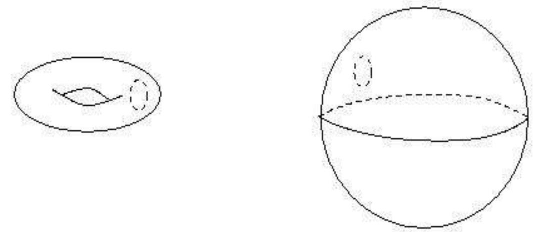

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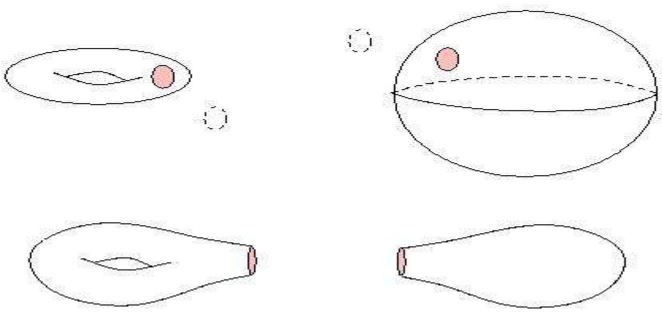

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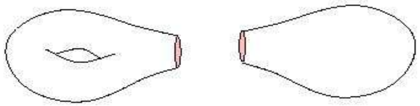

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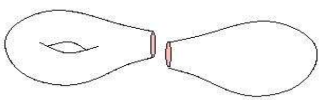

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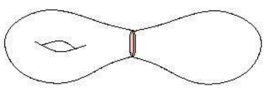

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Connected sum

of two surfaces is obtained by removing an open disk from each and then gluing the remaining spaces along their boundaries.

$S_1 \# S_2$ is the connected sum of S_1 and S_2

Question

- When you remove an open disk from a surface, do you get a surface back?
  - No, you get a surface with boundary. To make it a surface you have to “fill” in the boundary.

Examples

In all examples demonstrate your claims with pictures.

- What is the connected sum of a two holed doughnut surface and a three holed doughnut surface?
- What is the connected sum of a sphere and the Klein bottle?
- What is the connected sum of S^2 and P^2?

Conjecture

- What is the connected sum of any surface S and the sphere?
  - It is the surface S.
- Why?

$P^2 \setminus B^2$

Perform the following steps and draw a picture for each one.

2) Draw a projective plane = disk w opposite edge points identified.
3) Remove a small disk from the center of P^2
4) Draw a horizontal diameter. Cut along it and label the edges that were one with the same label.
5) Straighten each curved piece into a rectangle.
6) Physically glue together two long edges with the same label. Do the same with the remaining edges with same labels.

What did you get?

Möbius band