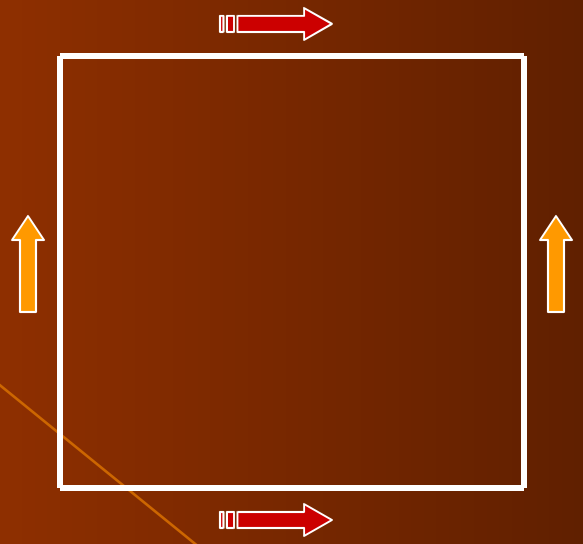
Flat Manifolds

Then the curved ones

Reminder

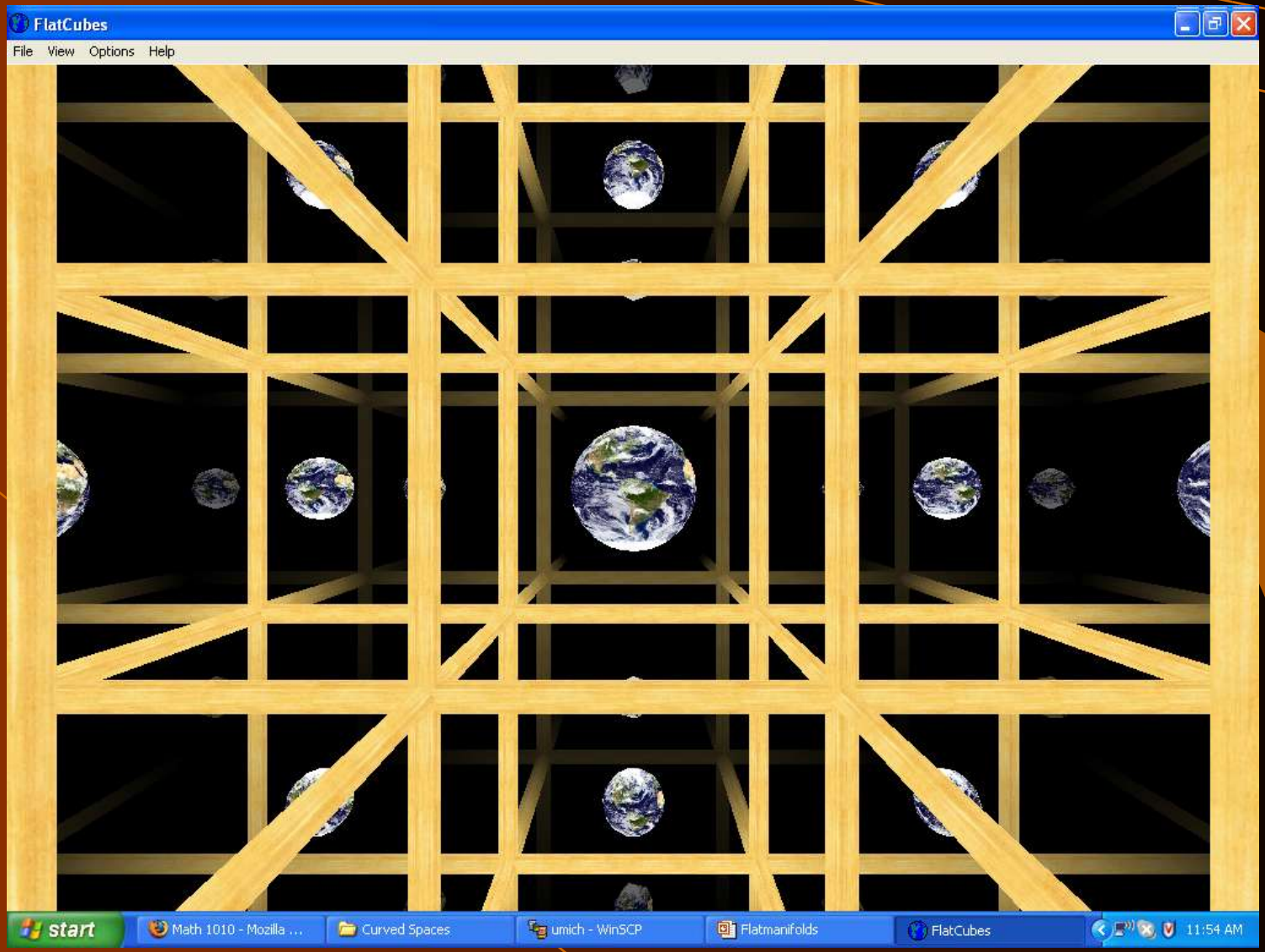


Fundamental domain

Tiling view

More

What is the tiling view of 3-torus?



Exercise 1

- Take the handout marked 1 and color the cubes according to the prescription.
  - What is the space you get?
- What do you get by coloring #2?

Exercise 2

- Take an unmarked handout and color it in correctly if the the sides of the cube are identified as follows:
  - Front to back with a side to side flip
  - Top to bottom with a side to side flip
  - Left to right normally

Homework for Wed.

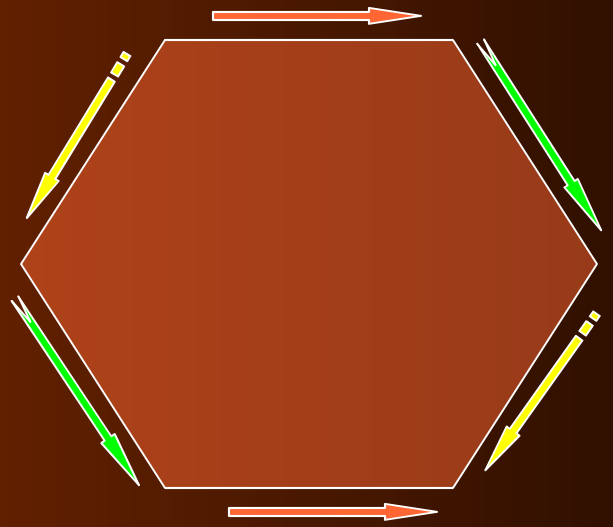
- Finish coloring handouts 3-6. If you are not given the gluing of the sides figure out what it is. In the handout 6 you are given the gluing, and you need to come up with the coloring.

Change

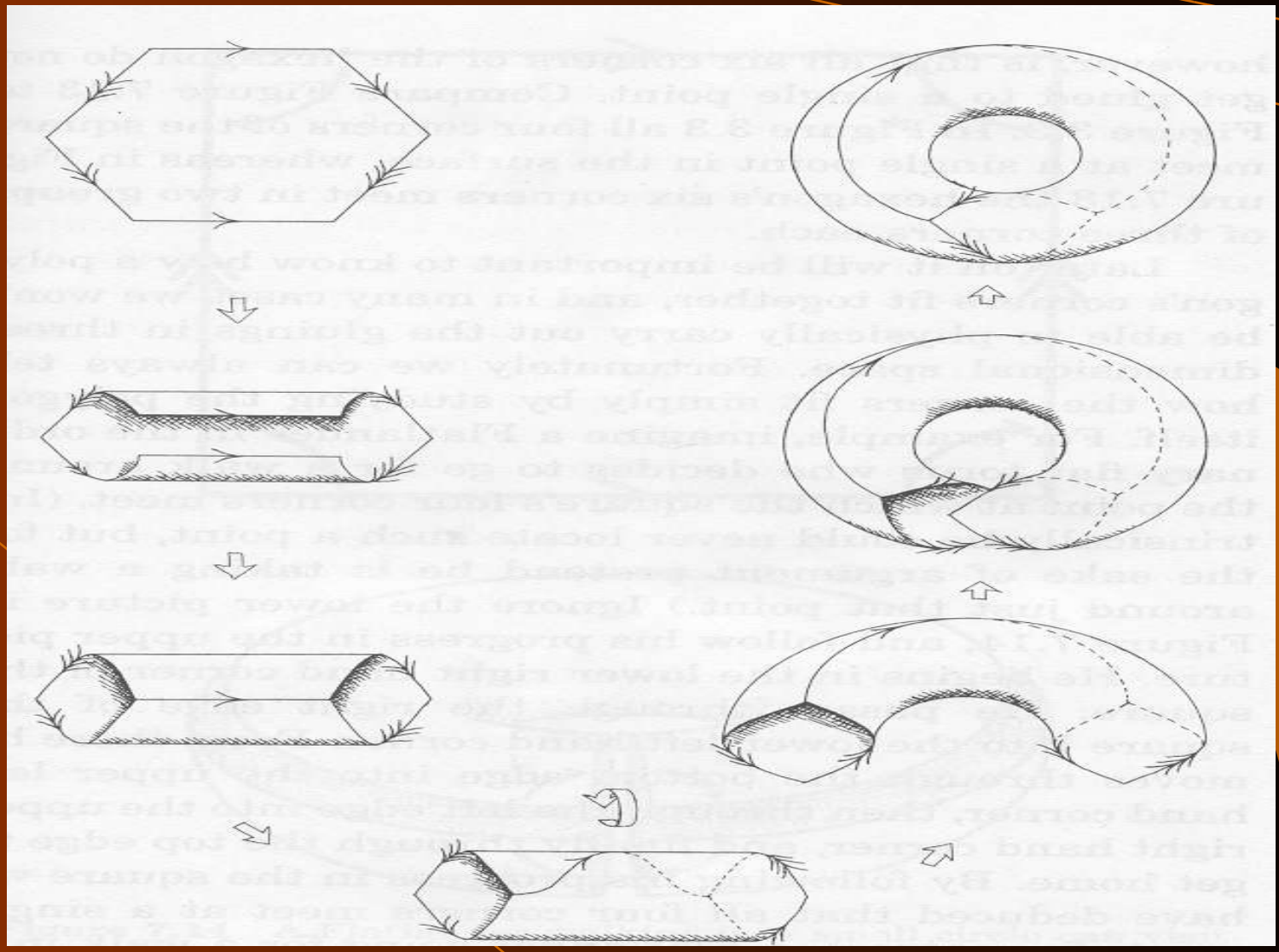
- Not all 3-manifolds are made from cubes
- Not all surfaces are made from squares.

Question

- What is:



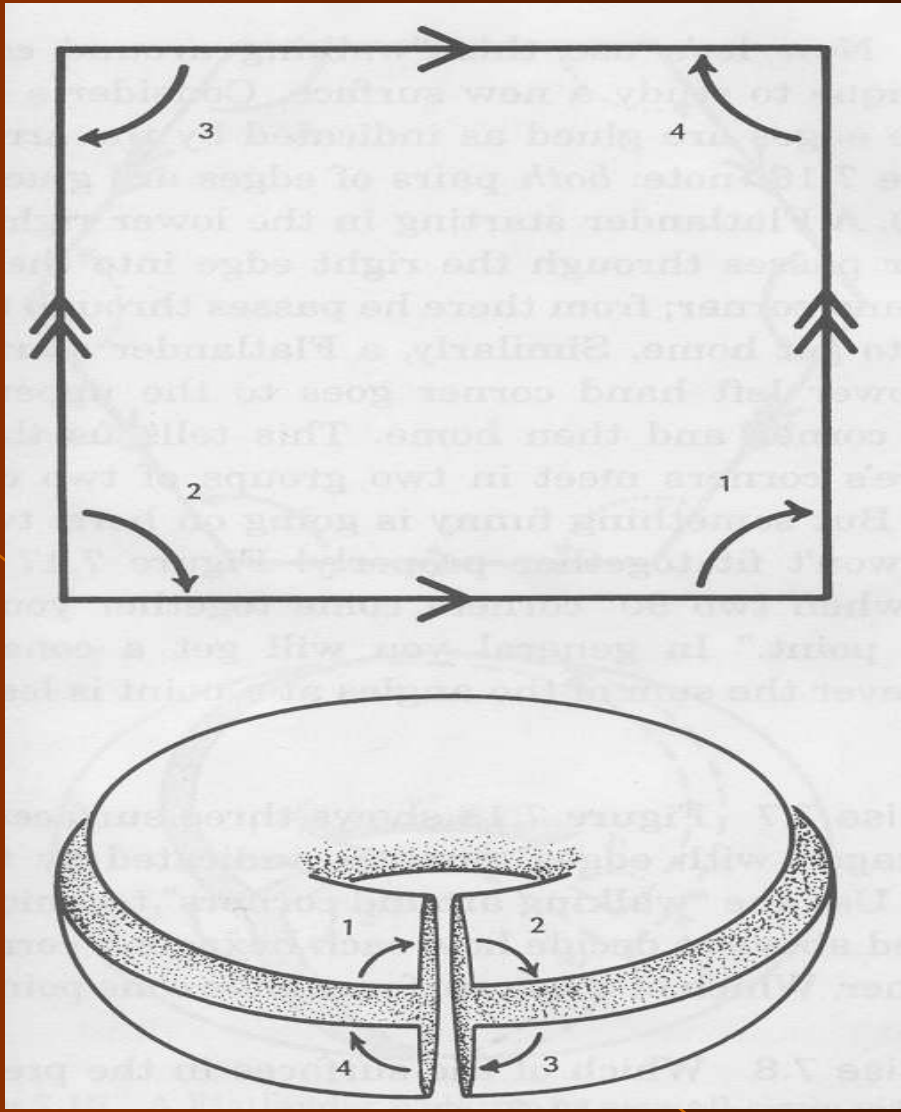
Answer



Investigation

- How are the corners fit together in the hexagonal torus, and how in a flat torus?

In flat torus



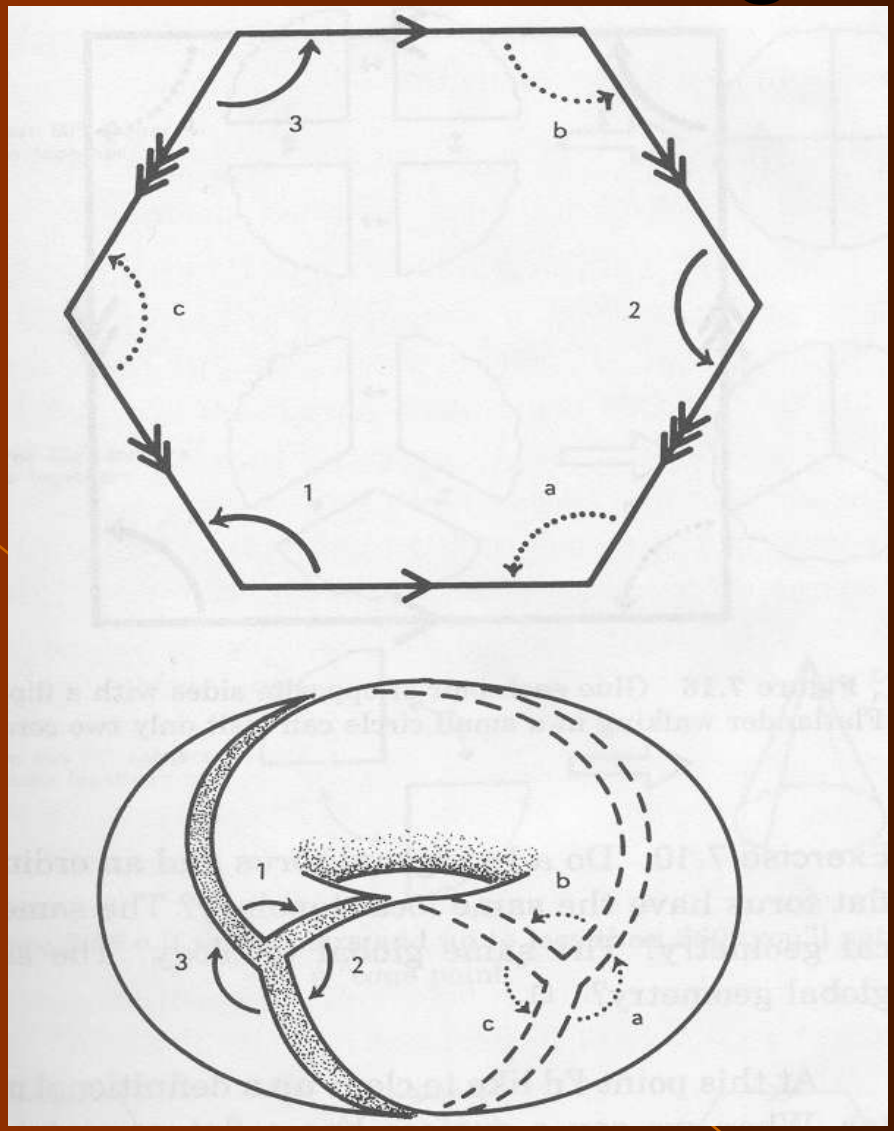
All four corners meet at one vertex

Exercise

- How big is each angle in a regular n-gon?

$$\frac{(n-2)\pi}{n}$$

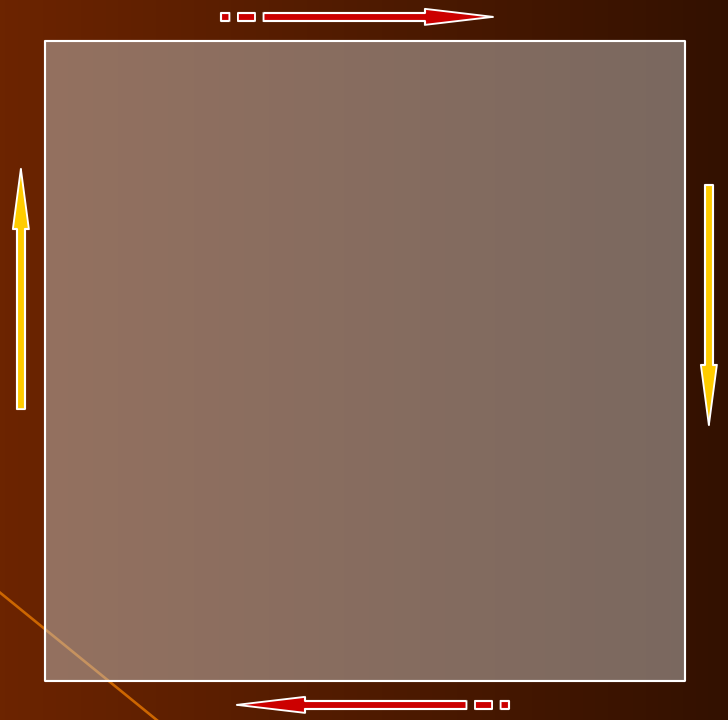
In hexagonal torus



Two groups of three corners meet each meeting at a single vertex

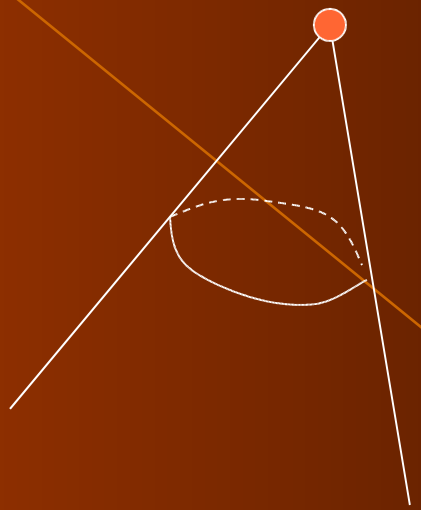
Problem

How do corners meet in this surface?



Answer

- In the projective plane we have two groups of two corners and two corners meet at a point:



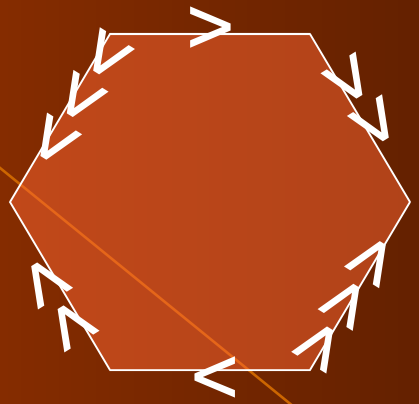
It can’t have flat geometry

Flat vs. nonflat surfaces

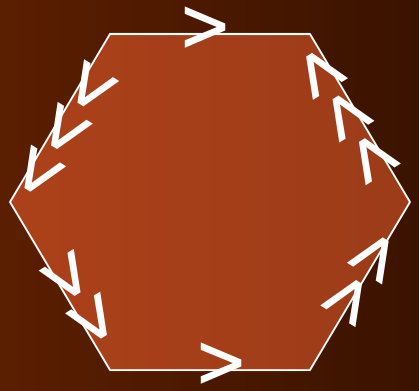
- If the angle sum of the corners that meet at a point is 360 degrees (and surface is flat away from the corners), then our surface has homogeneous flat geometry.
- If the angle sum of the corners that meet at a point is smaller than 360 degrees, the surface can not have a homogeneous flat geometry.

Exercise

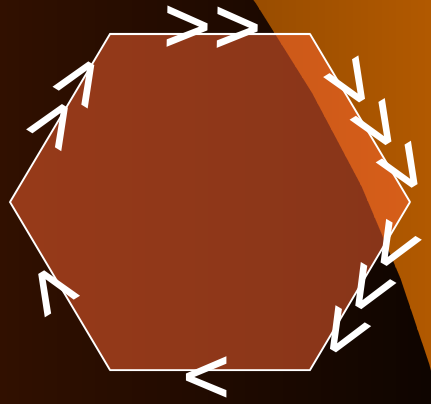
- Which of the following surfaces have cone points?



3 cone points



no cone points



opposite of cone points

More questions

- Are any of the surfaces in the previous exercise orientable?
- If these surfaces don't have a flat geometry, how will we find out what kind of geometry they do have?