Geometries on surfaces

Reminder

- Say  is a triangle with angles α, β and γ. If  lives on a
  - plane then $\alpha + \beta + \gamma = \pi$
  - sphere then $\alpha + \beta + \gamma > \pi$, and its area is $\alpha + \beta + \gamma - \pi$.

Question

- Is there a space in which  can live so that $\alpha + \beta + \gamma < \pi$ and so that its area will be $\pi - (\alpha + \beta + \gamma)$?
  - Yes.
  - Hyperbolic plane

What is hyperbolic plane?

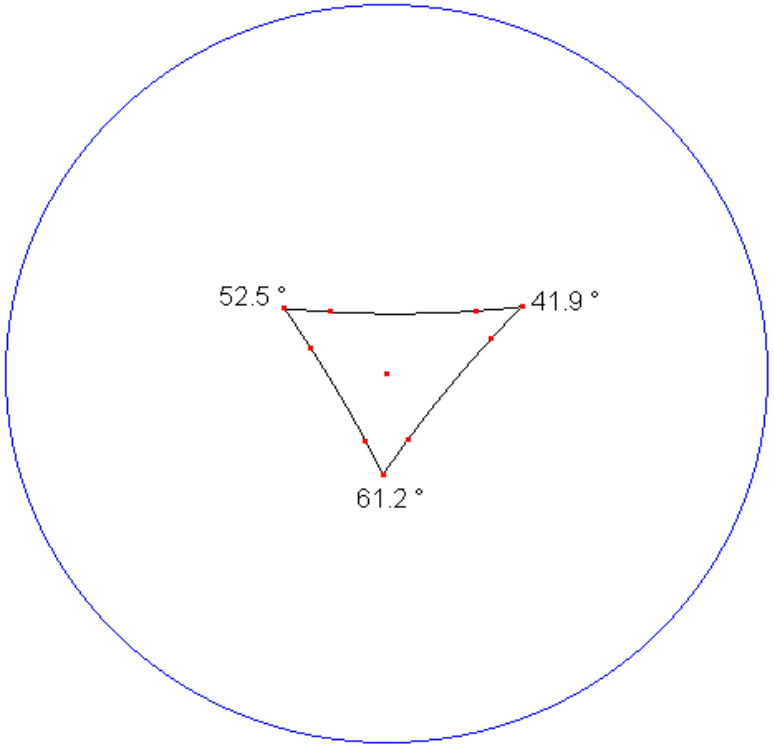
Your model of H^2

MODEL 1
http://www.geom.uiuc.edu/~crobles/hyperbolic/hypr/modl/uhp/
http://www.geom.uiuc.edu/~crobles/hyperbolic/hypr/modl/pncr/

Cabri Geometry II Plus - [Figure #5 *]

File Edit Options Session Window Help

1.



What is the sum of the angles in this triangle?

Cabri Geometry II Plus - [Figure #2 *]

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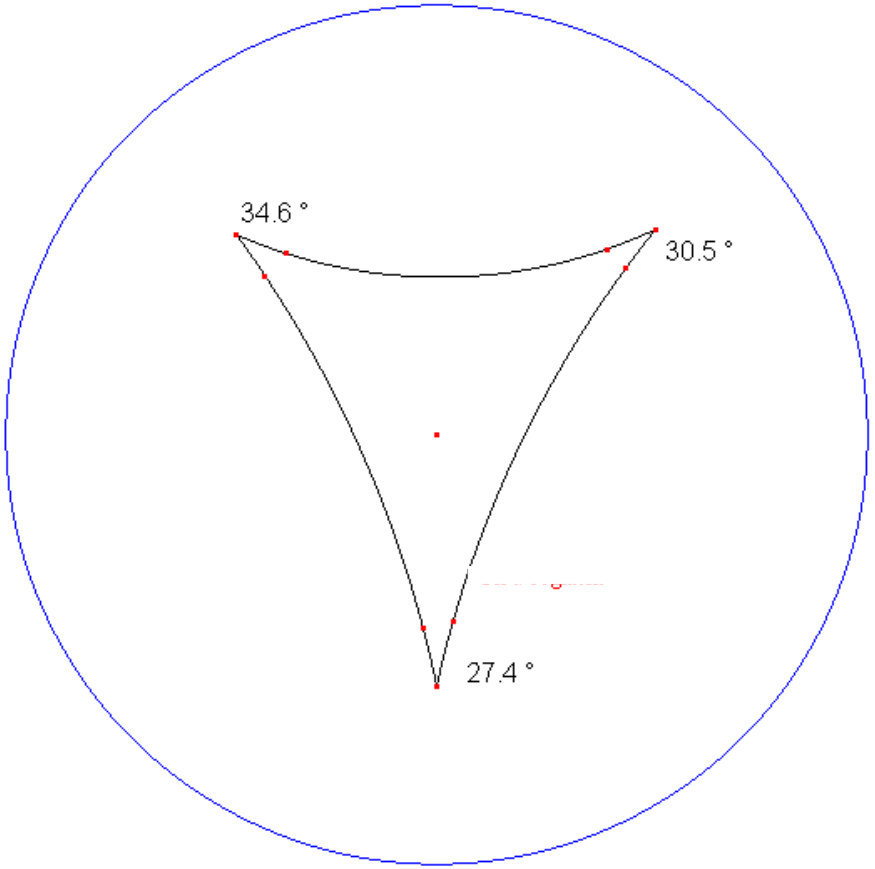
2.



Angle sum?

Cabri Geometry II Plus - [Figure #3 *]

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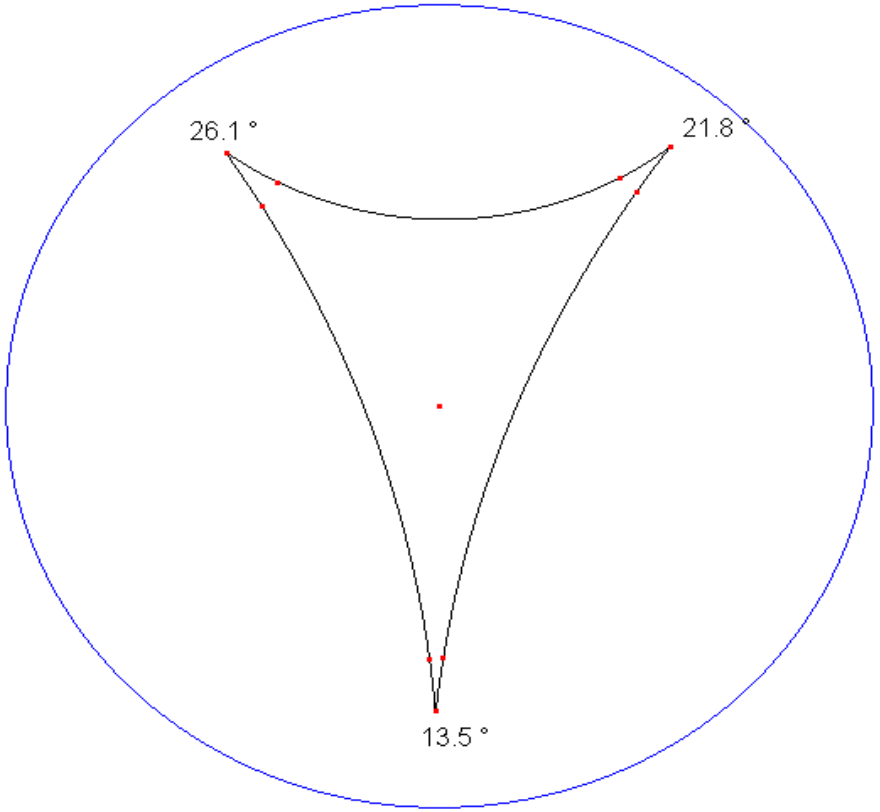
3.



Angle sum?

Cabri Geometry II Plus - [Figure #1 *]

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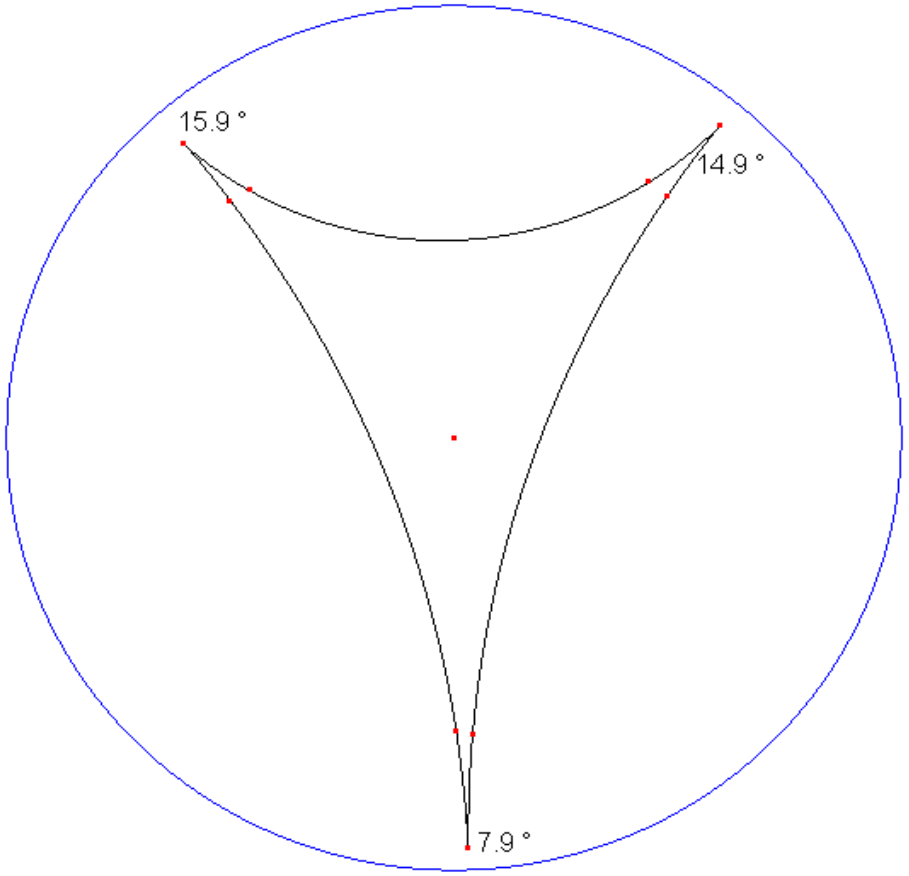
4.



Angle sum?

Cabri Geometry II Plus - [Figure #3 *]

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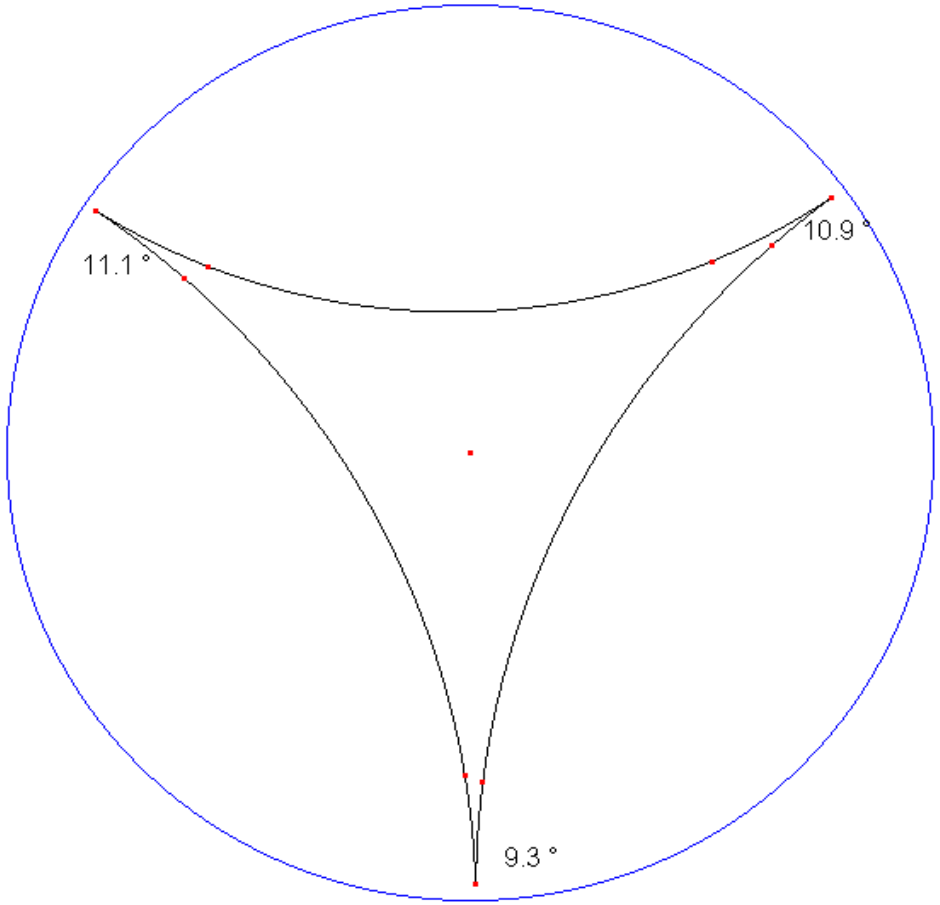
5.

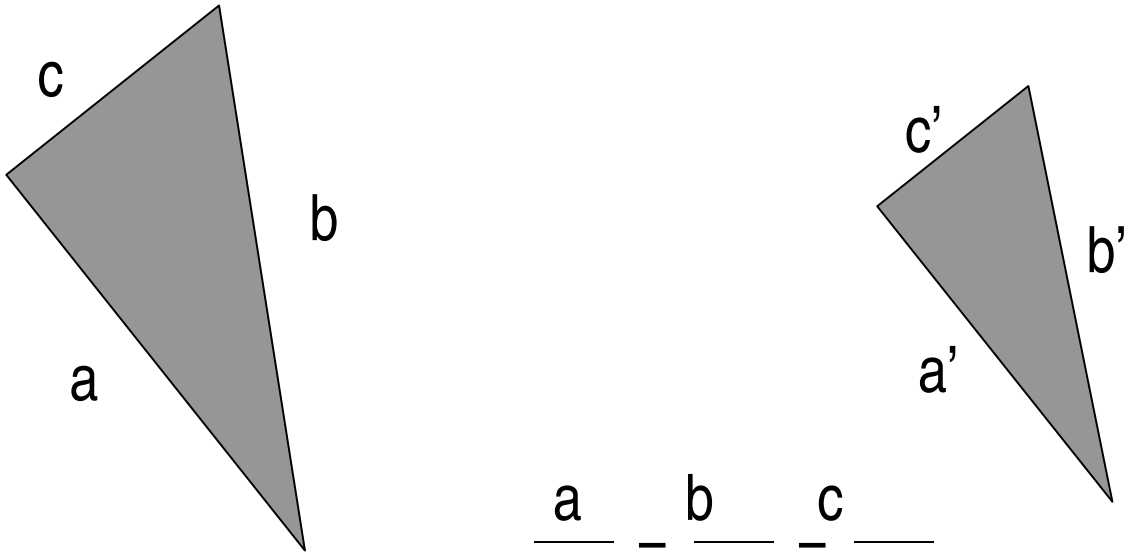


Angle sum?

http://www.cs.unm.edu/~joel/NonEuclid/NonEuclid.html

Definition

- We will say that two triangles are *similar* if their sides are proportional.



$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

Similar triangles in E^2, S^2 and H^2

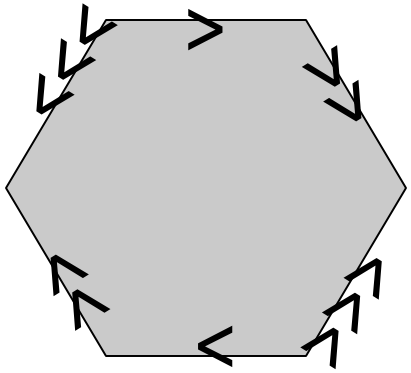
Q: What can you say about the angles of similar triangles?

In E^2: They remain the same

In S^2: As the sides increase, the angles increase

In H^2: As the sides increase, the angles decrease

Exercise 1



Each vertex is surrounded by two corners whose angles add up to 240°.

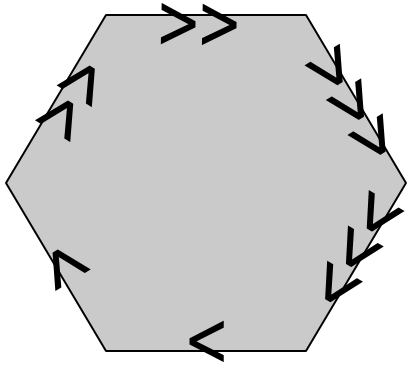
Find a hexagon whose angles are BIGGER then 120°.

Where will you look?

Sphere!!!!

Put a hexagon on a sphere and let it grow until each angle is 180°

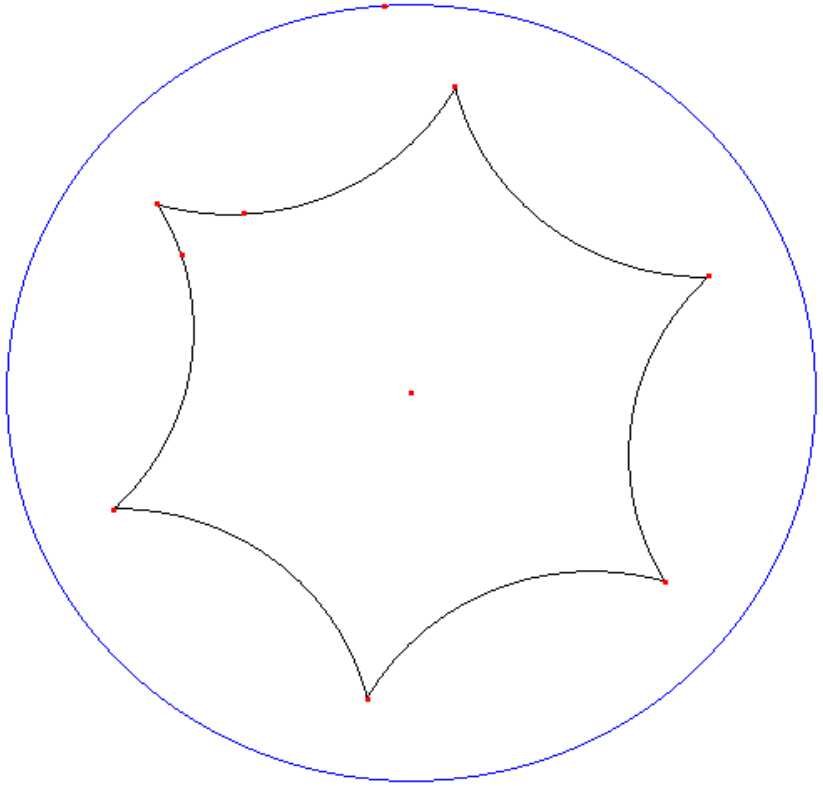
Exercise 2



The vertex is surrounded by corners whose angles add up to 720°

Find a hexagon whose angles are smaller than 120°

Draw a regular hexagon on hyperbolic plane and let it grow until its angles are 60°



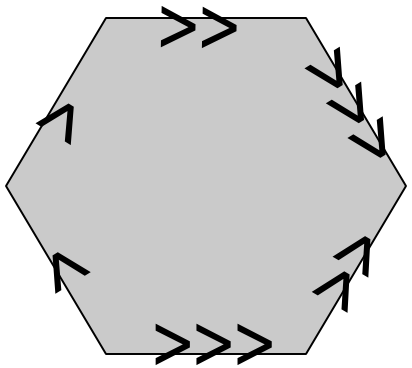
Every surface can be given some homogeneous geometry. If you can give a surface certain homogeneous geometry we say that surface admits that geometry:

- Admits elliptic (spherical) geometry
- Admits Euclidean geometry
- Admits hyperbolic geometry

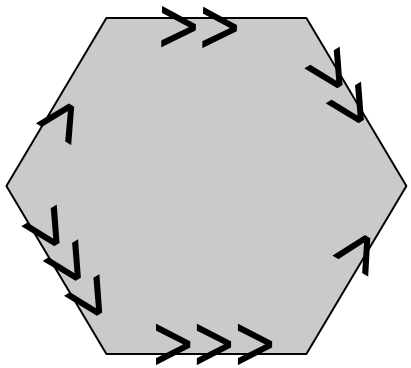
Exercise 3

In the following examples decide what type of geometry the surface admits and whether the surface is orientable or not.

a



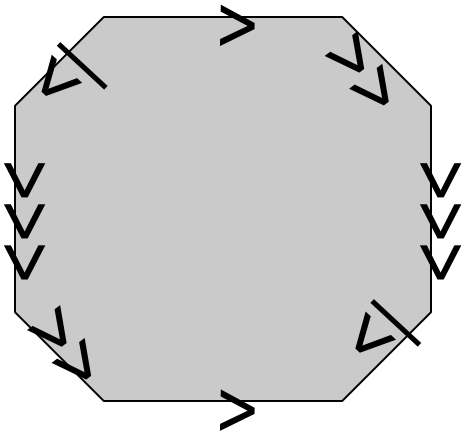
b



Exercise 3

In the following examples decide what type of geometry the surface admits and whether the surface is orientable or not.

c



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