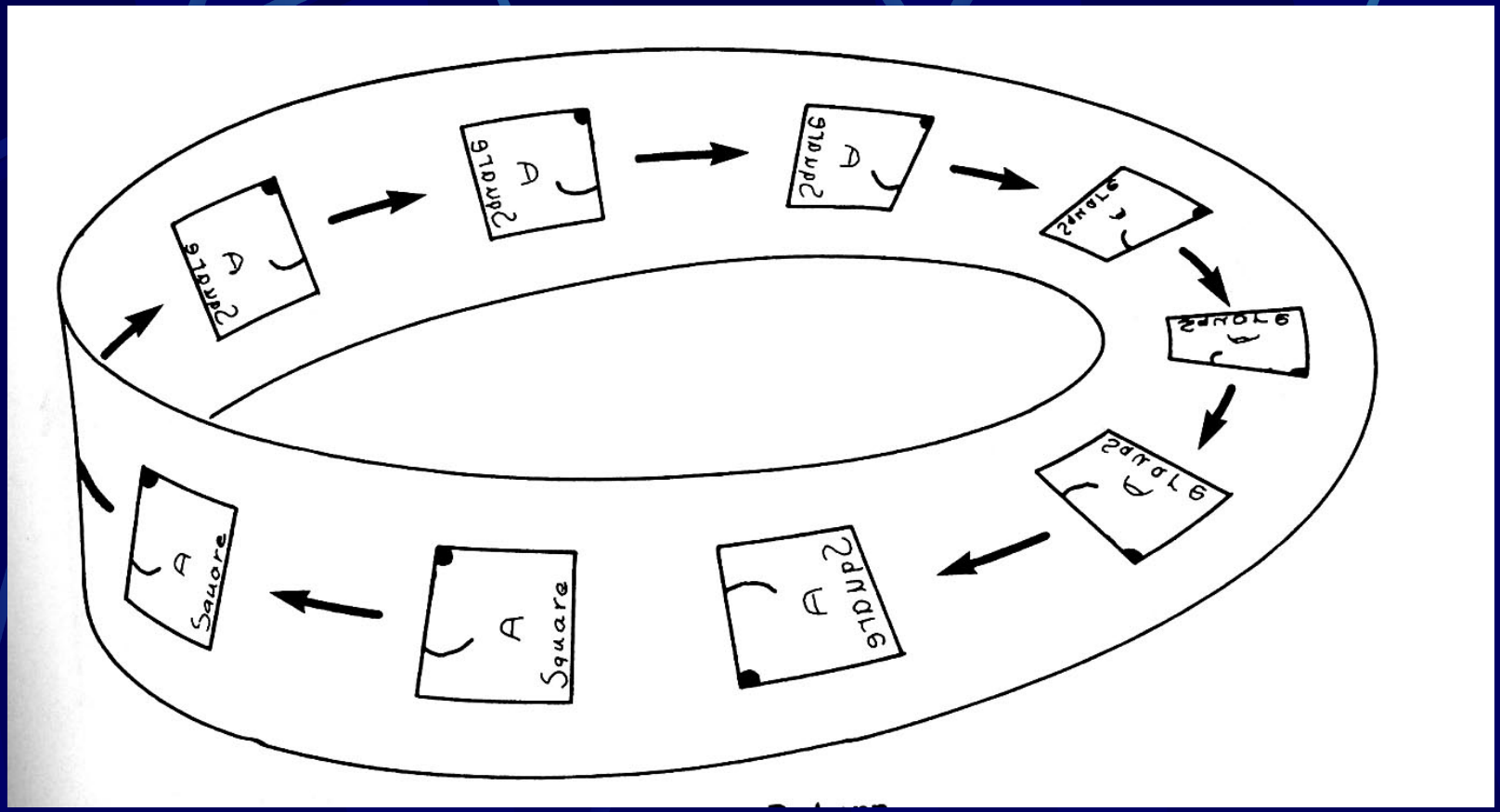
- Orientability

Story of Flatland once again

Welcome to Flatsbourgh

Part of the Flatland



Whole Flatland?

- What sort of manifold could contain such crazy piece of land?
  - What 2-MANIFOLD (surface) could contain a Möbius band?

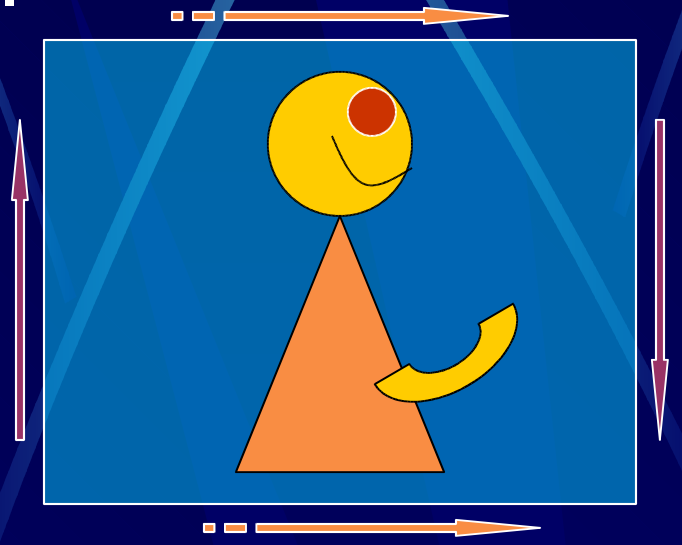
Klein bottle

Exercise 1

- Give at least three examples of Möbius band inside a Klein bottle.

Exercise 2

- This is the fundamental domain view of the Klein bottle.



Draw the tiling view!

Play

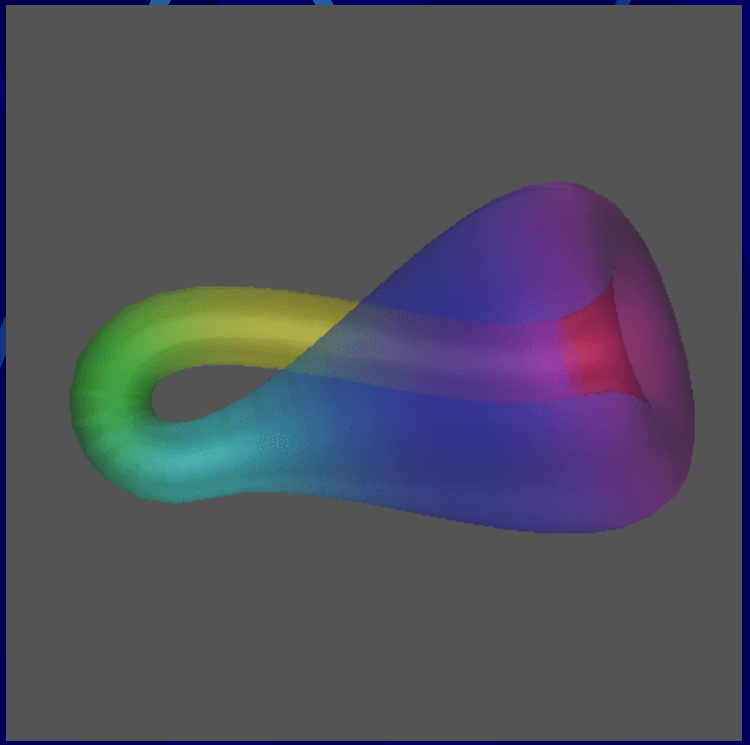
http://www.geometrygames.org/TorusGames/index-old.html

Exercise 3

- What is local geometry of the Klein bottle?
  - Homogeneous
  - Flat

Klein bottle in 3-space

- Can you glue the gluing diagram?
  - one way
  - another way



Different pictures

- the view
- in the making

Orientability

Def: A path in a manifold that brings a traveler to her original position mirror reversed is called an *orientation reversing path*.

Def: Manifold that do not contain orientation reversing paths are called *orientable*, and the ones that do are called *nonorientable*.

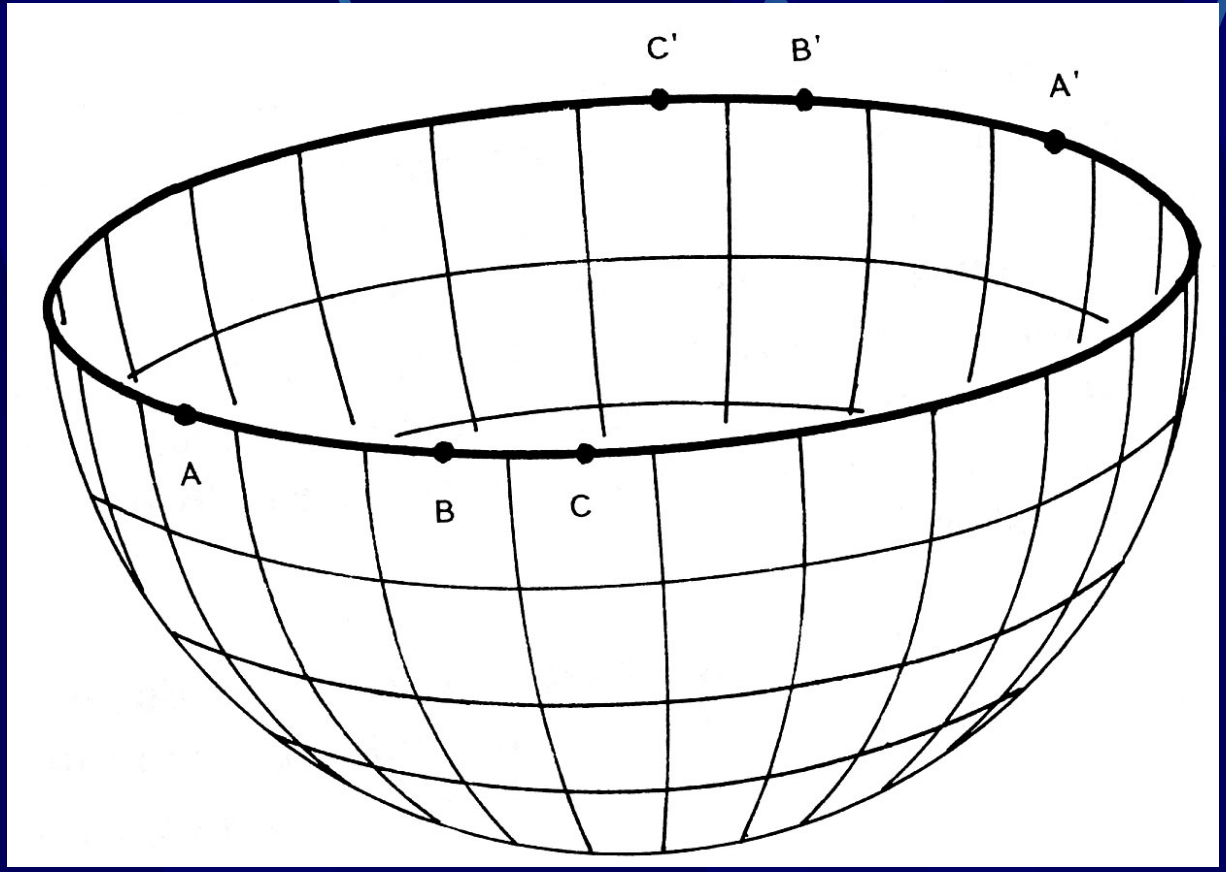
Exercise 4

1. Is orientability an intrinsic or extrinsic property?
2. Give examples of orientable manifolds.
3. Give examples of nonorientable manifolds.
4. Can you make a nonorientable 3-manifold?

Nonorientable 3-manifold

- What do you see as you turn around in this manifold?

Projective plane

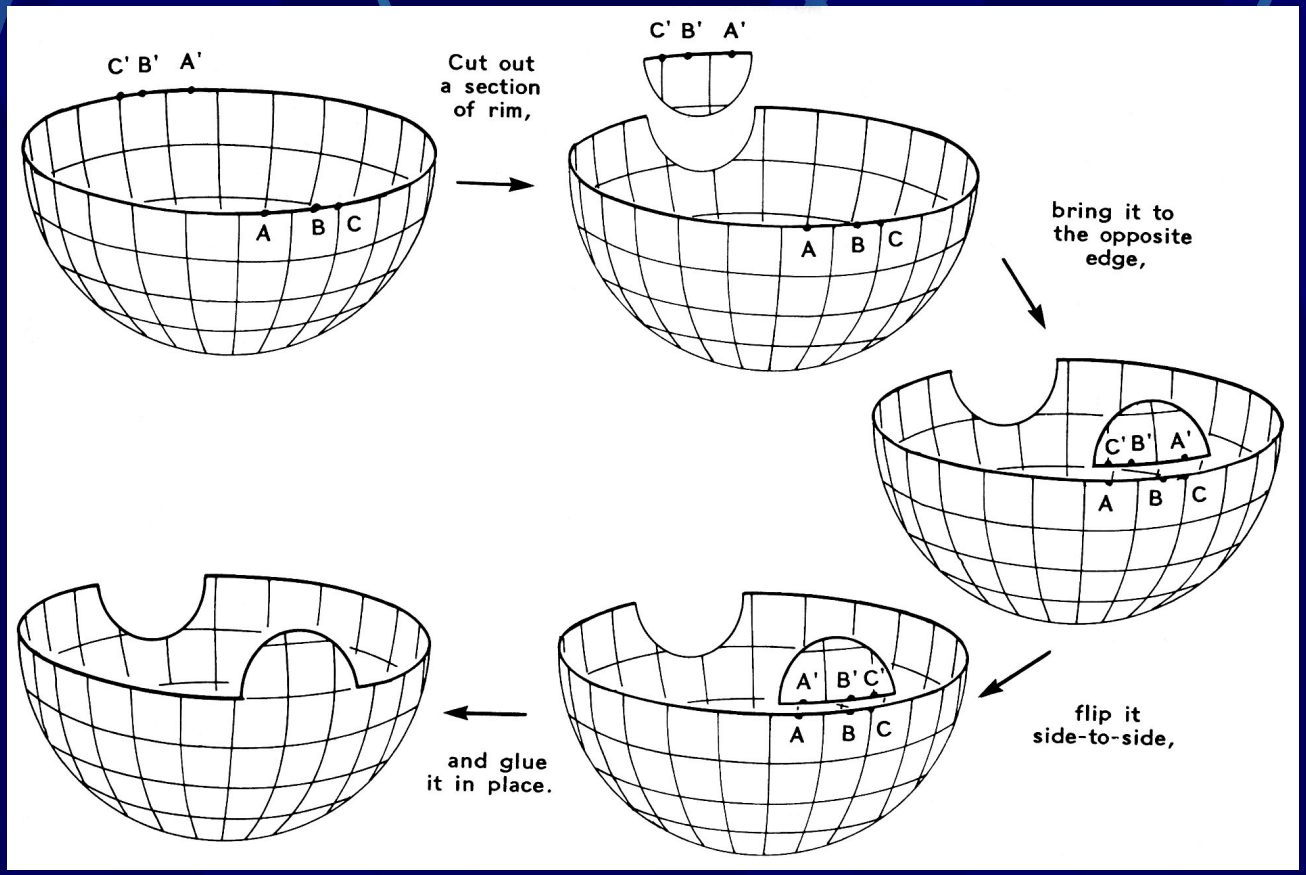


Def : Projective plane P^2 is a space obtained from a hemisphere by identifying the opposite points on its rim.

Exercise 5

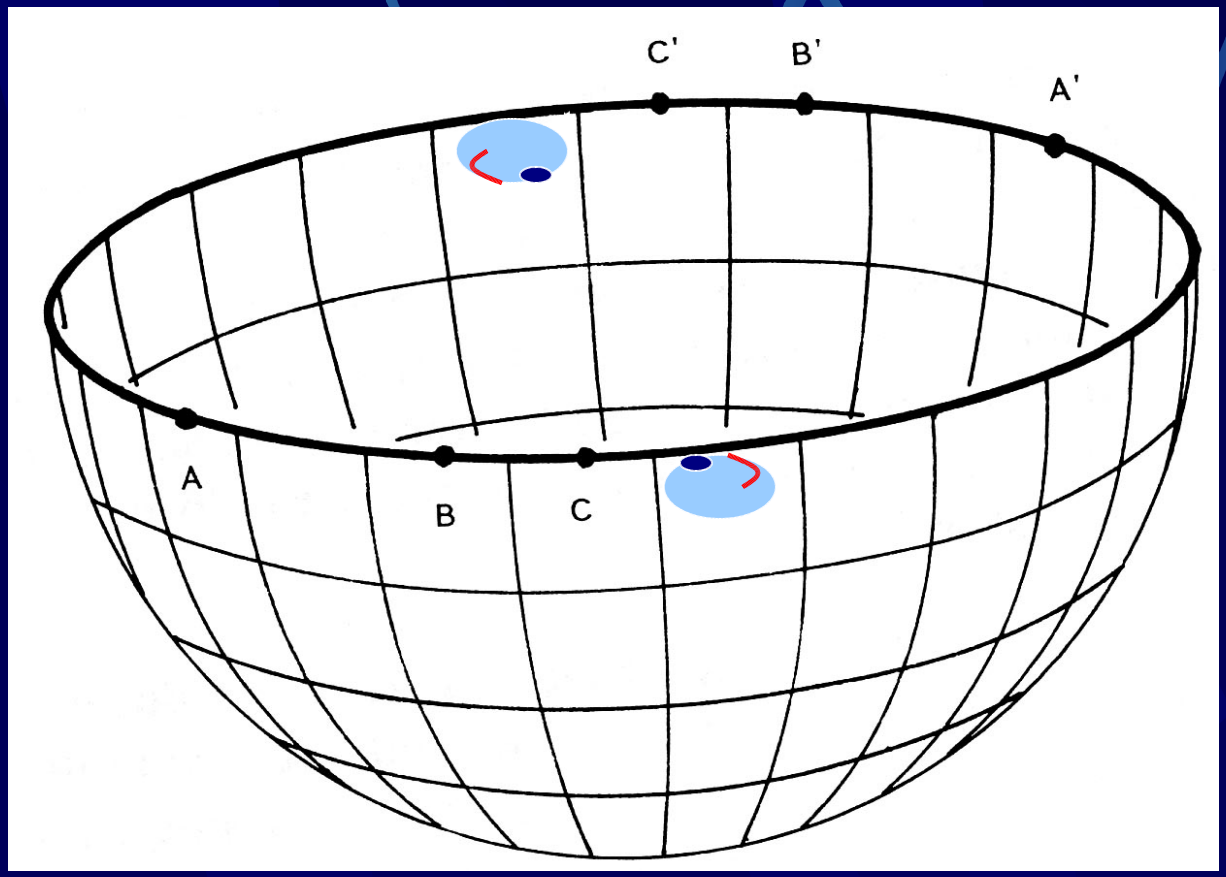
- Is the projective plane flat or curved?
  - curved
- Is the projective plane homogeneous?
  - yes

Local picture in P^2

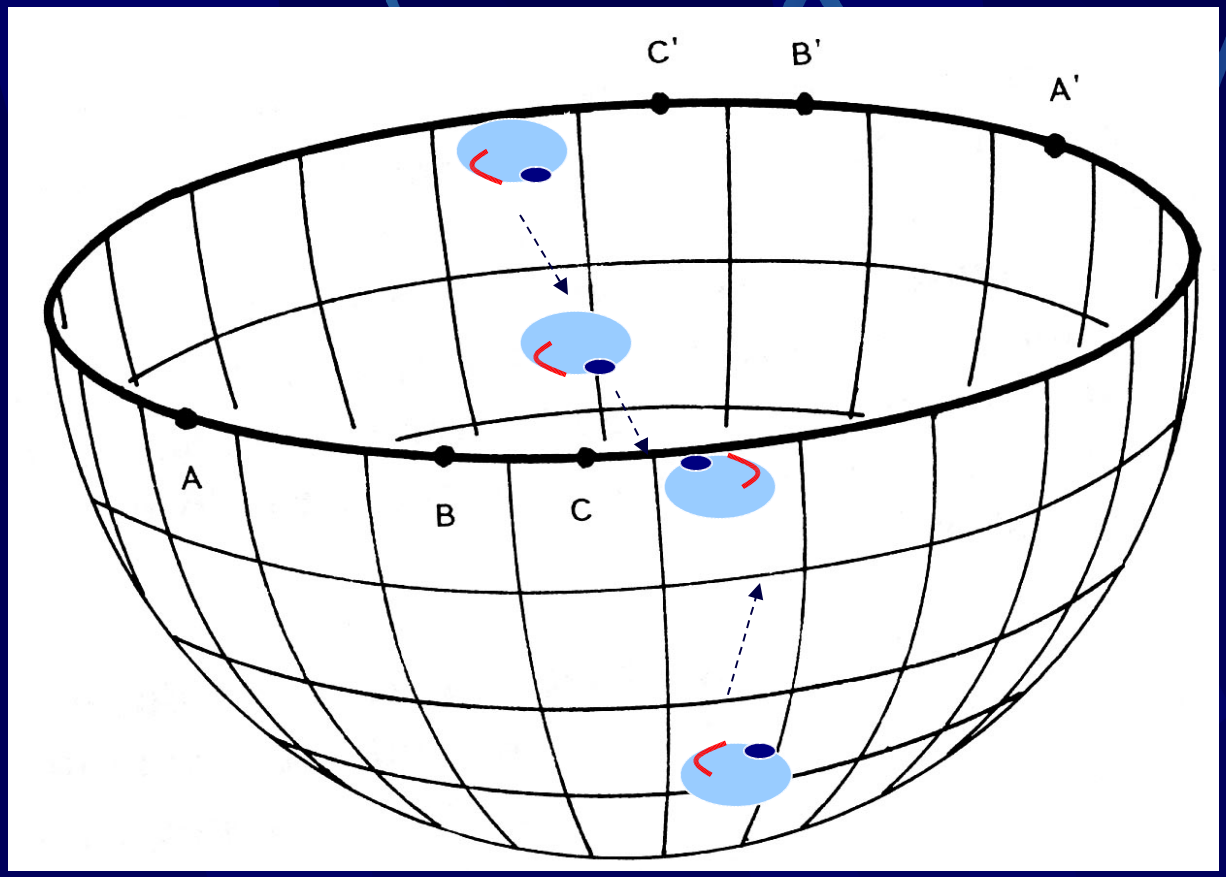


Exercise 6

- Is the projective plane orientable or not?



We found
an orientation
reversing path



Nonorientable

Exercise 6

- A Flatlander lives on a projective plane. His house is at the south pole. One day he leaves his house and walks in a straight line until he returns home. When was he farthest away from home?

Exercise 7

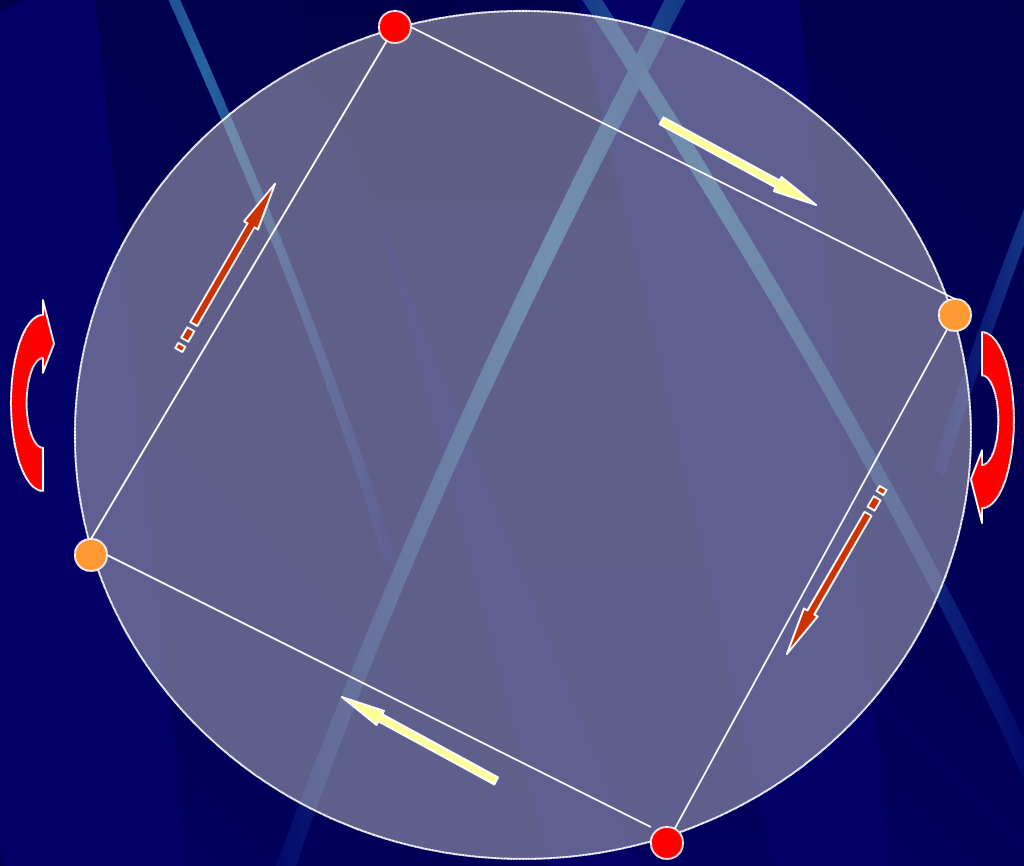
- Two fire stations are to be built on a projective plane. Where should they be built so that they are most efficient (as far as possible)? What if three fire stations are to be built?

Fill in the table with manifolds

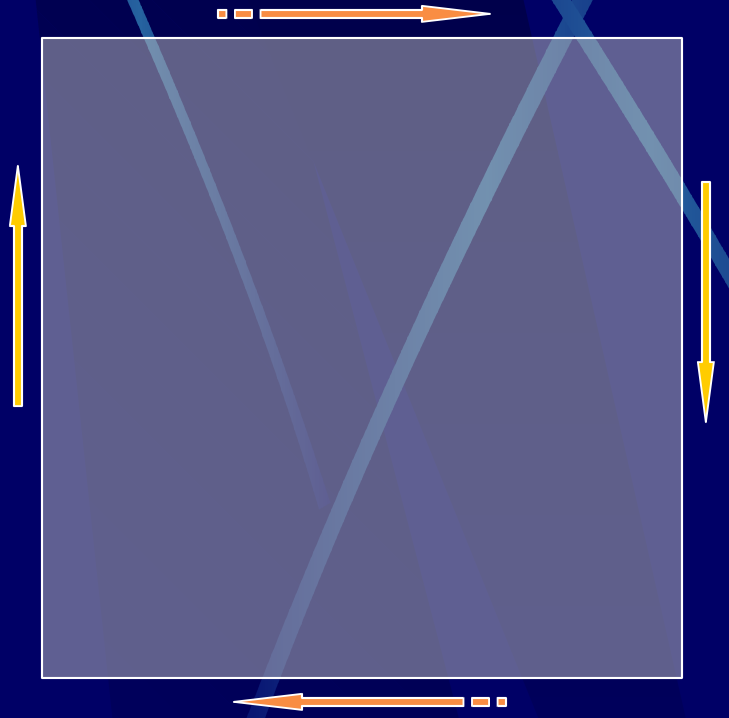
	Orientable	Nonorientable
Curved local geometry		
Flat local geometry		

Exercise 8

- Which space that you know is topologically same as a hemisphere?



Projective plane



Exercise 1

- Find Möbius bands inside the projective plane.
- Can you really glue the gluing diagram of the projective plane inside our 3-space?

Different pictures of P^2

Cross cap

roman surface

boy's surface

Exercise 2

- How would you make a projective 3-space?
- Is the projective 3-space orientable?
- Is orientability a local or global property? Is it topological or geometrical?