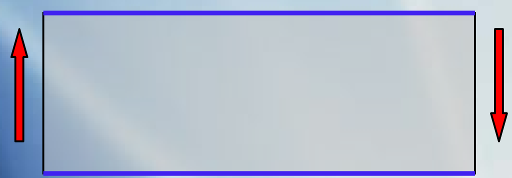
Connected sums

And Products

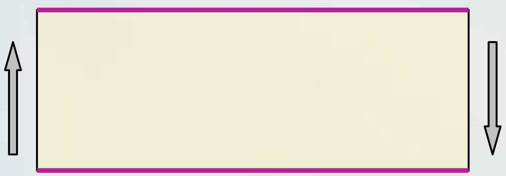
$P^2 \# P^2 = ???$

Take two copies of $P^2 \setminus B^2$ and glue along their boundaries. Make sure to remember what was the boundary of the disk you cut out from P^2.

Solution

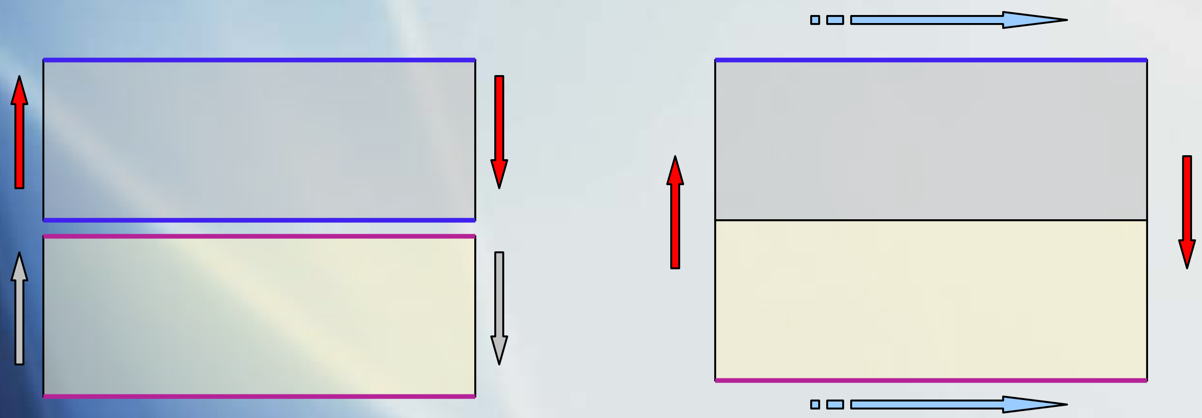


$P^2 \setminus B^2$



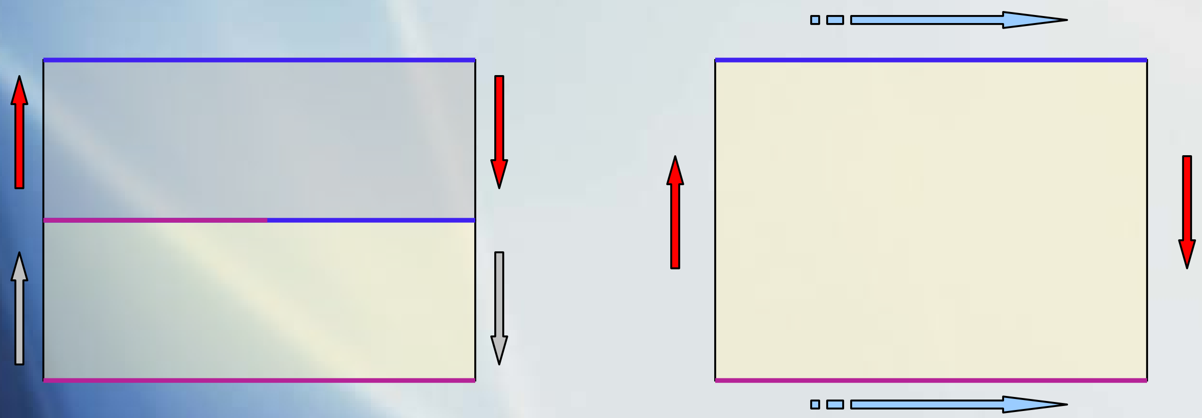
$P^2 \setminus B^2$

Solution



Klein bottle K^2

How are these two spaces the same?



Big Theorem

Every surface is a connected sum of tori and/or projective planes.

All surfaces

# of pp → / # of tori ↓	0	1	2	3
0	S^2	P^2	$P^2 \# P^2$	$P^2\#P^2\#P^2$
1	T^2	$T^2 \# P^2$	$T^2\#P^2\#P^2$	
2	$T^2 \# T^2$	$T^2\#T^2\#P^2$	….	
3	$T^2\#T^2\#T^2$			….

Exercise

- Find a surface on the list that is topologically equivalent to
  - $K^2 \# P^2$
  - $K^2 \# T^2$
  - $K^2 \# K^2$

Question one should ask

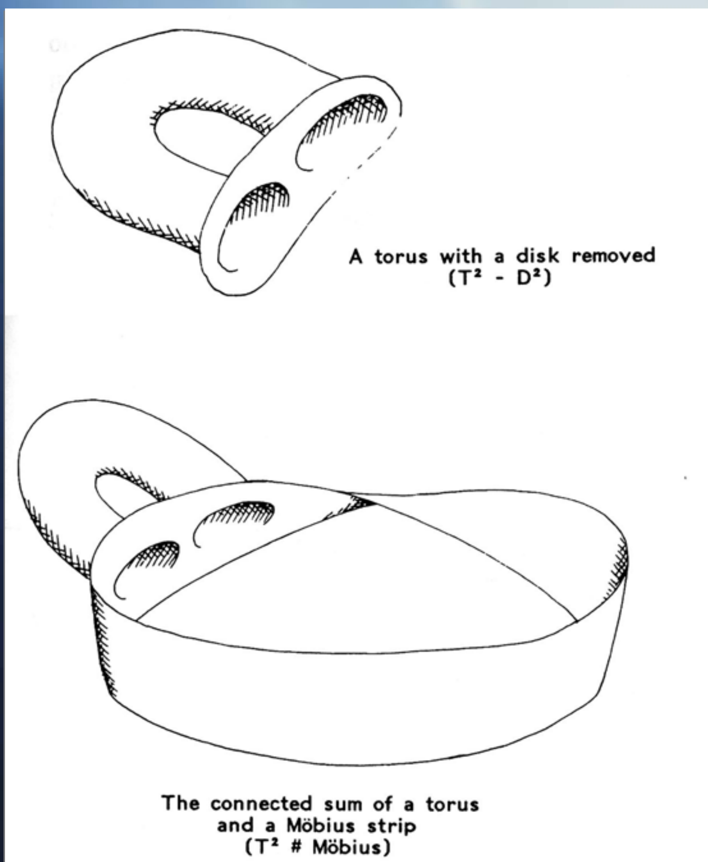
- Does this list contain duplicates?
- Yes.

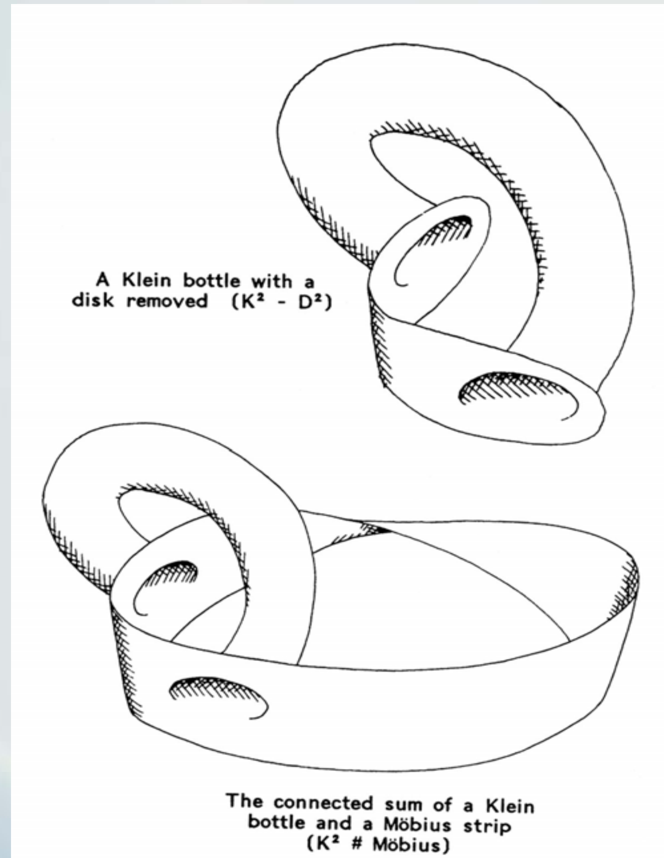
Investigation

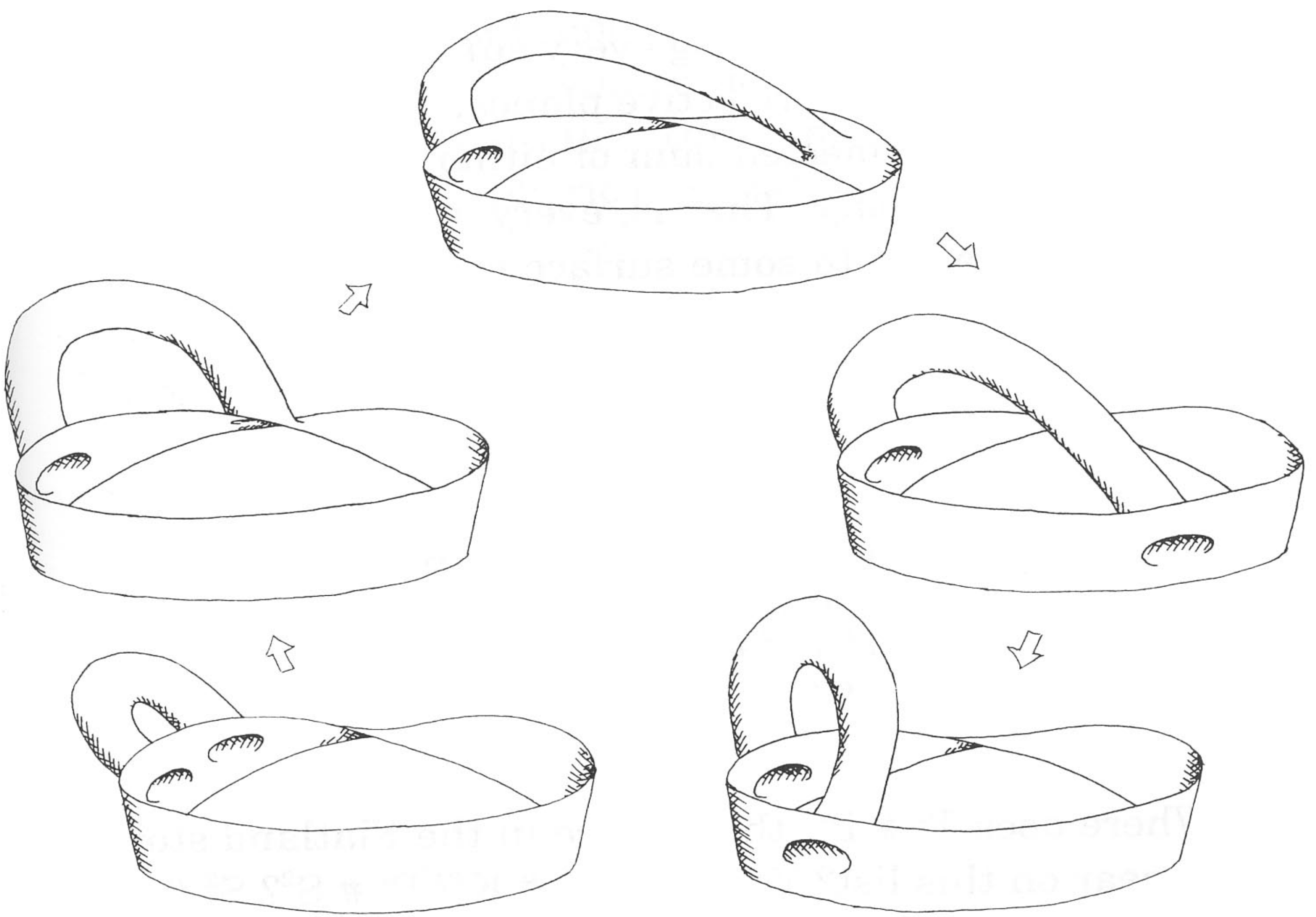
I would like us to consider these two spaces

$T^2 \# P^2$ and $K^2 \# P^2$

Study the pictures







Exercise

Show that

$$T^2 \# P^2 = P^2 \# P^2 \# P^2$$

Exercise

- Why can our table from few slides ago be replaced by this table:

S^2

T^2 P^2

$T^2 \# T^2$ $P^2 \# P^2$

$T^2 \# T^2 \# T^2$ $P^2 \# P^2 \# P^2$

and so on

Exercise

- Match the entries from Column A with the same entries in Column B:

Column A	Column B
$S^2 \# T^2$	$P^2 \# P^2$
K^2	$K^2 \# P^2$
$S^2 \# S^2 \# S^2$	$P^2 \# P^2 \# P^2 \# K^2$
$P^2 \# T^2$	$S^2 \# S^2$
$K^2 \# T^2 \# P^2$	T^2

Exercise

- Which of the surfaces on the list are orientable and which are nonorientable?
  - The connected sums of tori only are orientable
  - The connected sums of pps only are nonorientable

Exercise

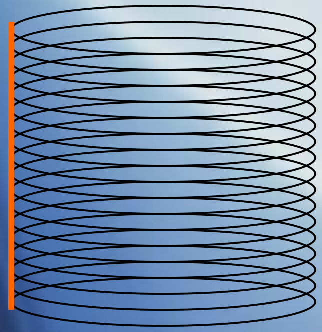
- How would you generalize the connected sum to 3-manifolds?
  - Take out an open ball from both manifolds and glue them back up along their spherical boundary.

Question

- If you make a connected sum of two spaces, what is the dimension of the resulting space equal to?
    - Same as the dimension of the spaces we started with.
- Could we form new manifolds from old, but so that the dimension changes?

Products

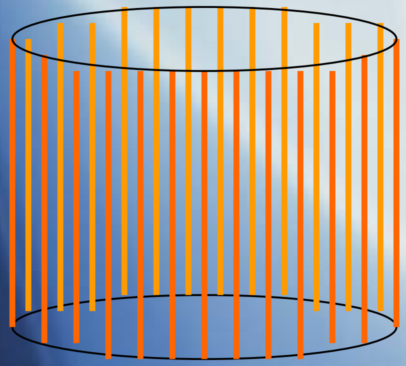
- Stack some circles!



Cylinder
-- interval of circles

Or maybe

- Stack some intervals!



Cylinder
-- circle of intervals

Cylinder

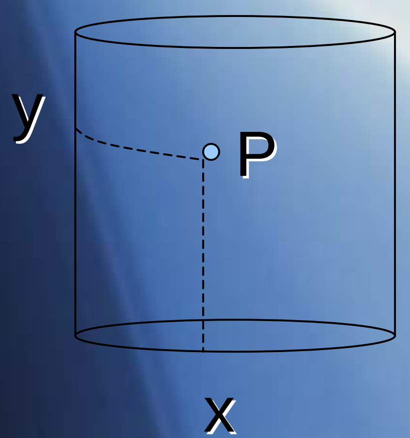
- Is a product of a circle and an interval

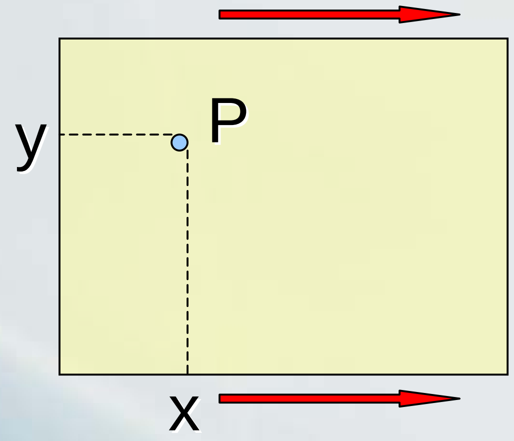
$$S^1 \times I$$

(circle cross interval)

In coordinates:

- Any point P on the cylinder can be given as (x,y), where x is a point on the circle and y is a point on the interval





Exercise

- Show on the gluing diagram of the cylinder its product structure.
  - Mark how it is an interval of circles
  - Mark how it is a circle of intervals