

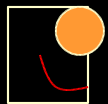
New spaces



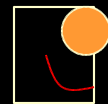
What is $S^2 \times S^1$?

We live in a 3-dim'l manifold – we want to SEE 3-dim'l manifold

compare to

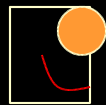


lives in a 2-dim'l
manifold



wants to see 2-dim'l
manifold

Become



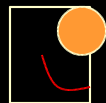
and try to imagine **$S^1 \times S^1$** !

Remember: torus is a product of a circle and a circle

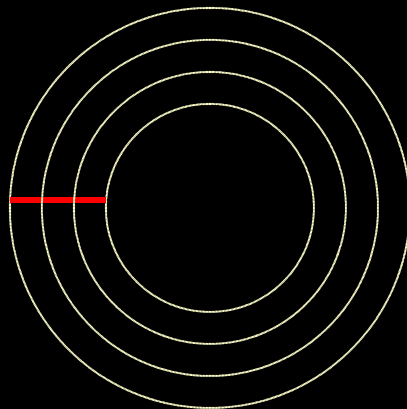
or

It is a cylinder whose top circle is glued to the bottom circle

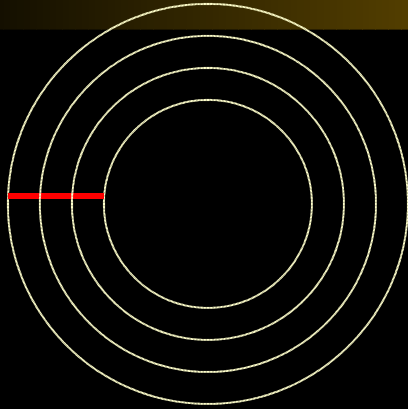
Q: How can
and an interval)?



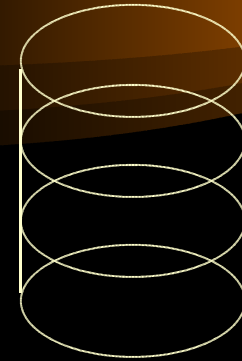
imagine a cylinder (product of a circle



In the plane



In the space



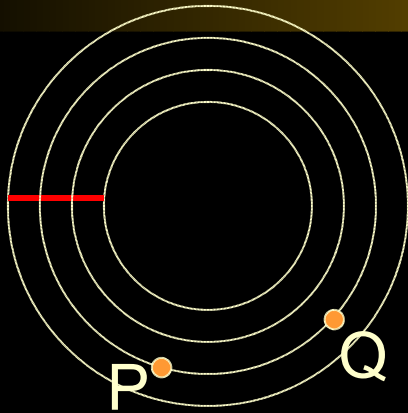
He's got a cylinder:

Glue the inner circle
to the outer circle

We've got a cylinder:

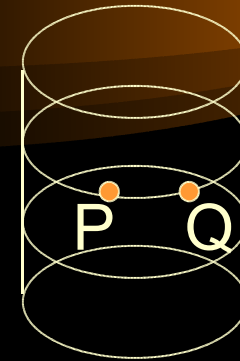
Glue the bottom circle
to the top circle

In the plane



What is the shortest path between P and Q?

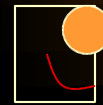
In the space



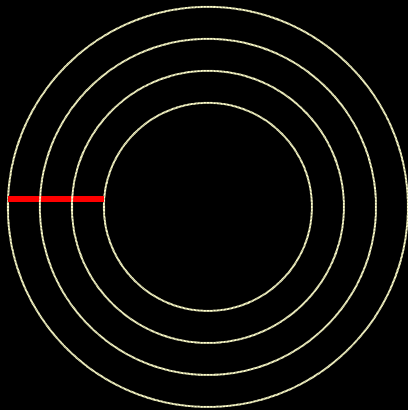
What is the shortest path between P and Q?

In each case along the circle between P and Q

To make this a geometric torus

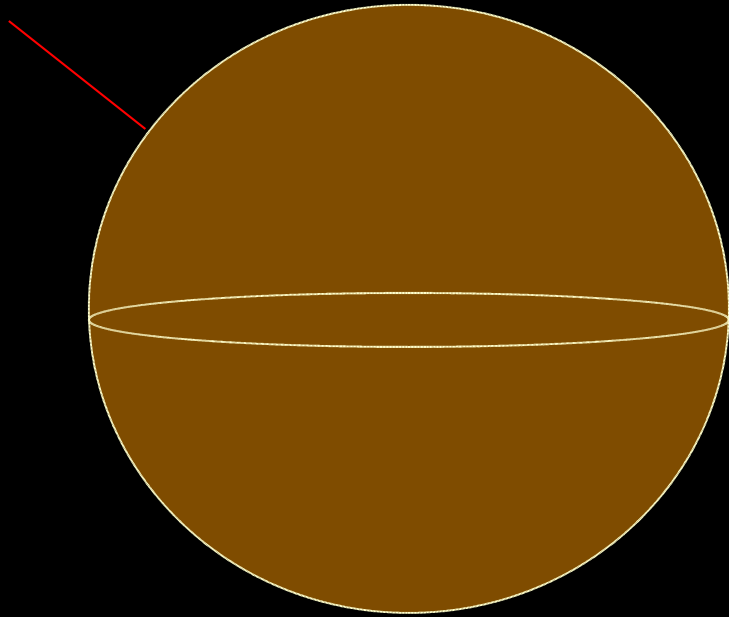


has to

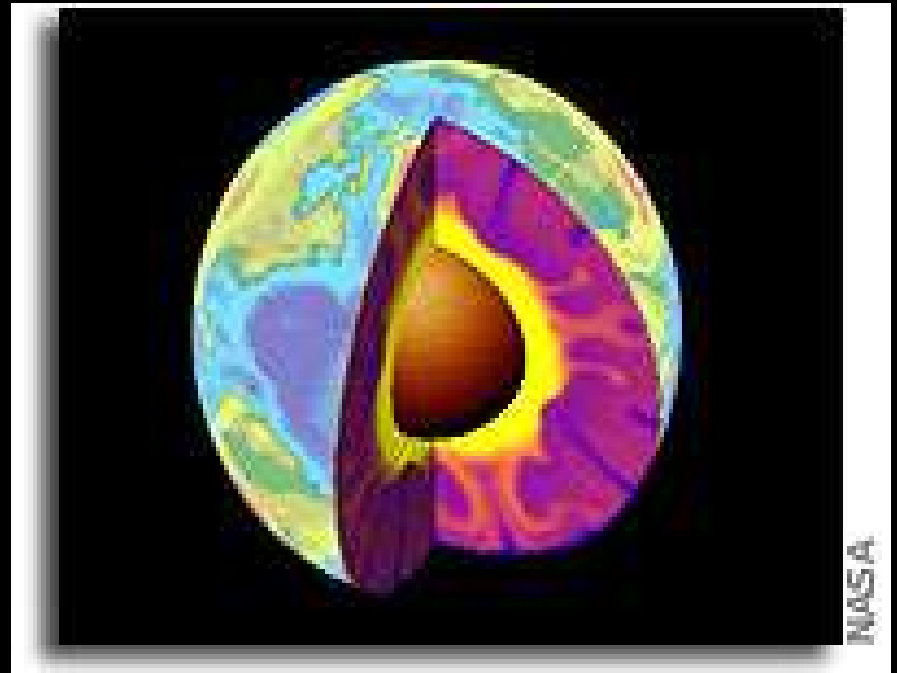


that all the circles are of the same size.

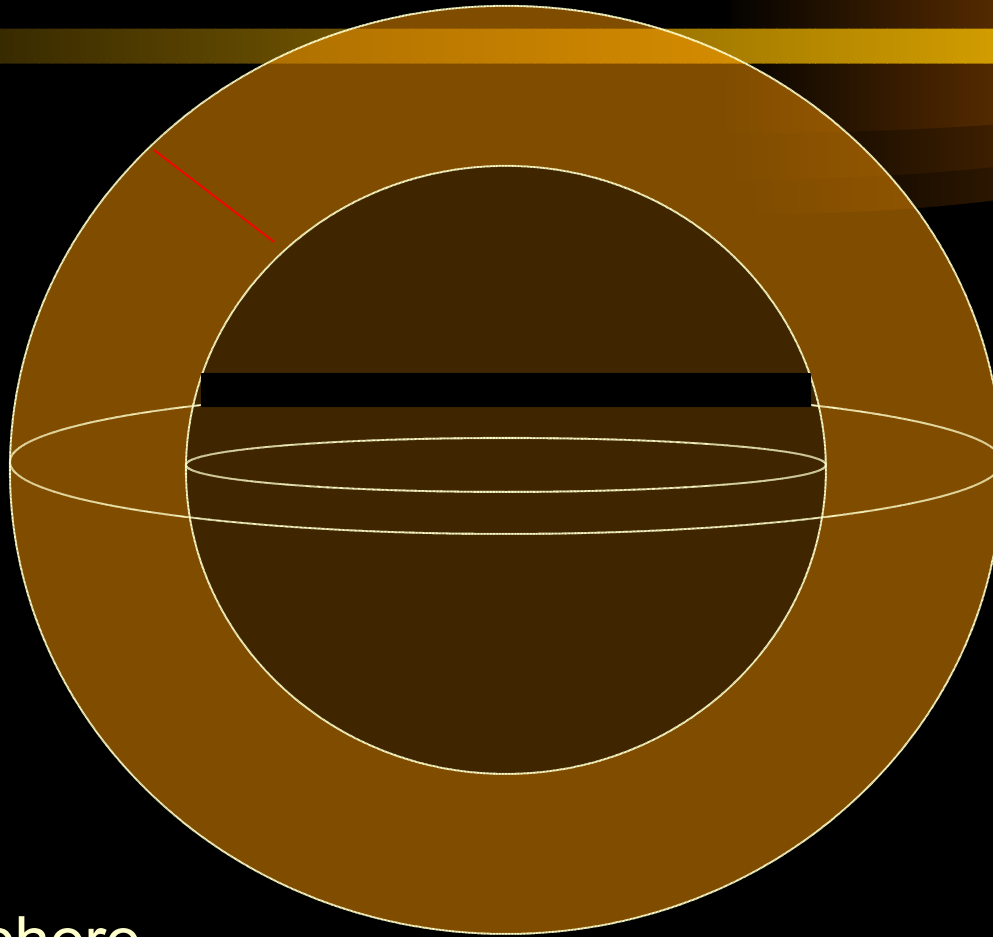
What is $S^2 \times S^1$?



First make interval of spheres



What is $S^2 \times S^1$?



Glue inner sphere
to outer sphere

Questions



- What did red interval become?
- What is the horizontal cross section?
- What are the other things you notice about this manifold?

Isotropic manifolds

- A manifold in which geometry is same in every direction is called *isotropic*.
 - Is $S^2 \times S^1$ isotropic?
 - Is $S^2 \times S^1$ homogeneous?

Exercises

- Are there any surfaces which are homogeneous, but not isotropic?
- What is a nonorientable 3-manifold that is a product and has the same local geometry as $S^2 \times S^1$?
- How would you imagine a product of surface of genus 2 with a circle?