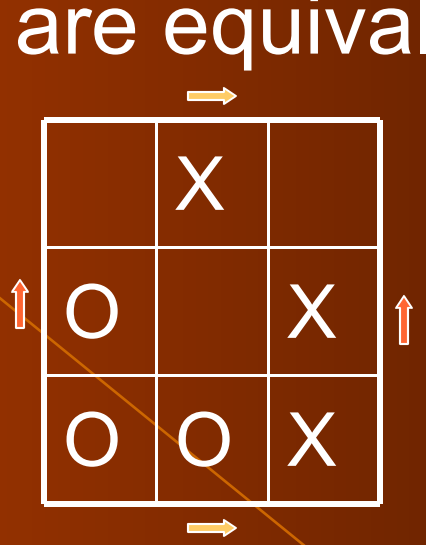
Tori cont'd and Vocabulary

Torus Tic Tac Toe

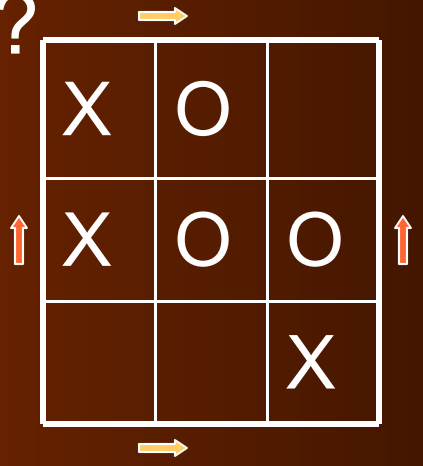
- Does the first move matter in torus tic-tac-toe?
- If the first player takes the upper left corner, how many nonequivalent moves does the second player have?

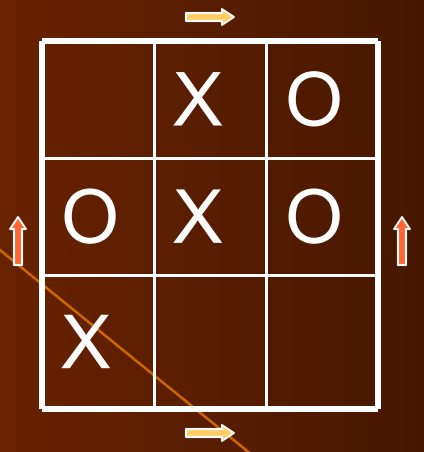
Exercise

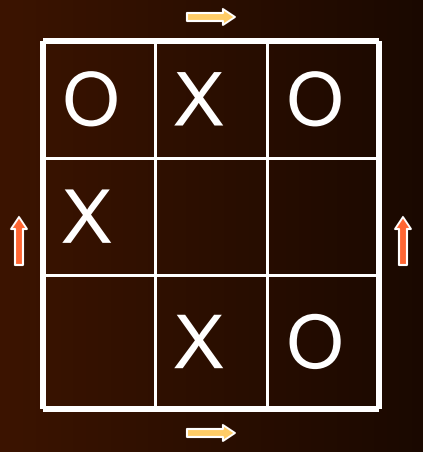
Which of the following torus tic-tac-toe games are equivalent?



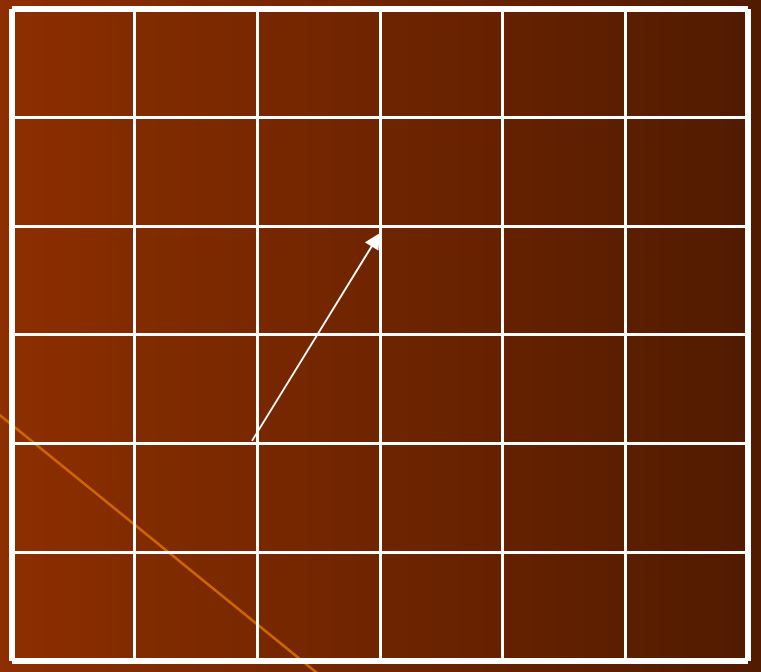
all







Exercise



Draw the path of a ladybug who walks in a straight line until she returns to her starting point if she always walks 2 units northward for each unit eastward.

View?

View?

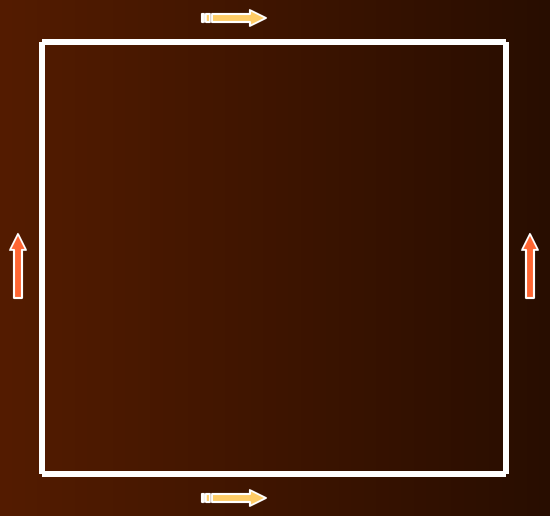
View?

View?

Tiling view

More

Fundamental domain

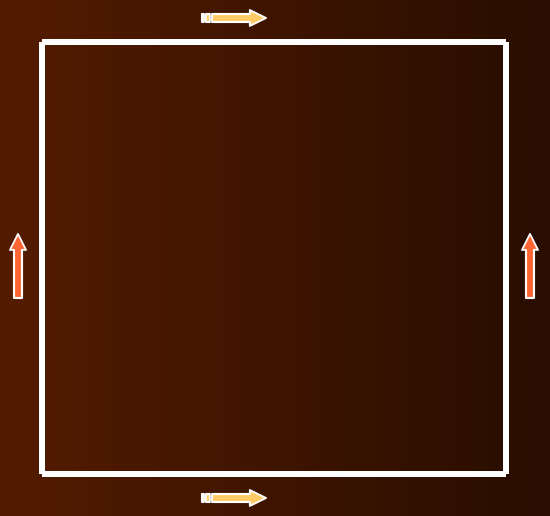


Tiling vs. Fundamental Domain

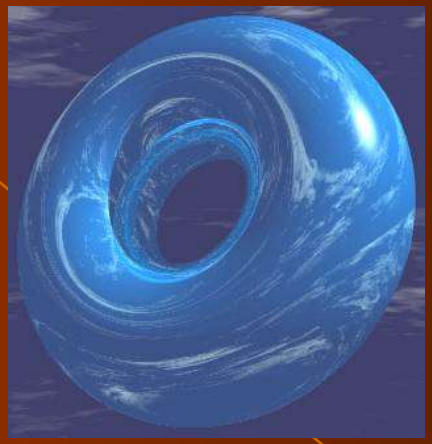
- Tiling view shows correctly that the torus has no boundary, but falsely suggests that it is infinite
- Fundamental domain view shows correctly that the torus is finite, but incorrectly suggests that it has a boundary.

Putting a torus into 3-space

cylinder of a cylinder one way
and another way
finally

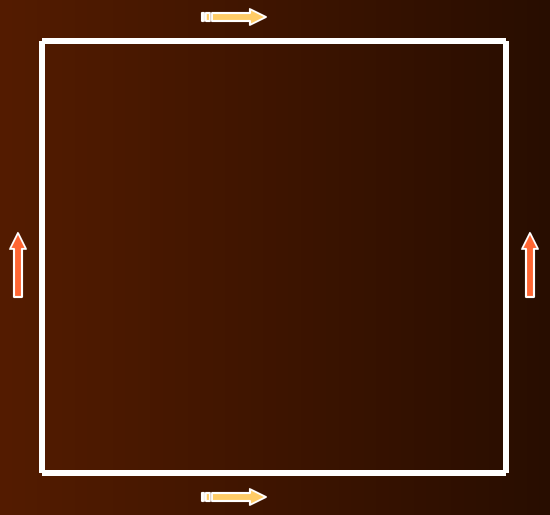


Torus vs. Flat Torus



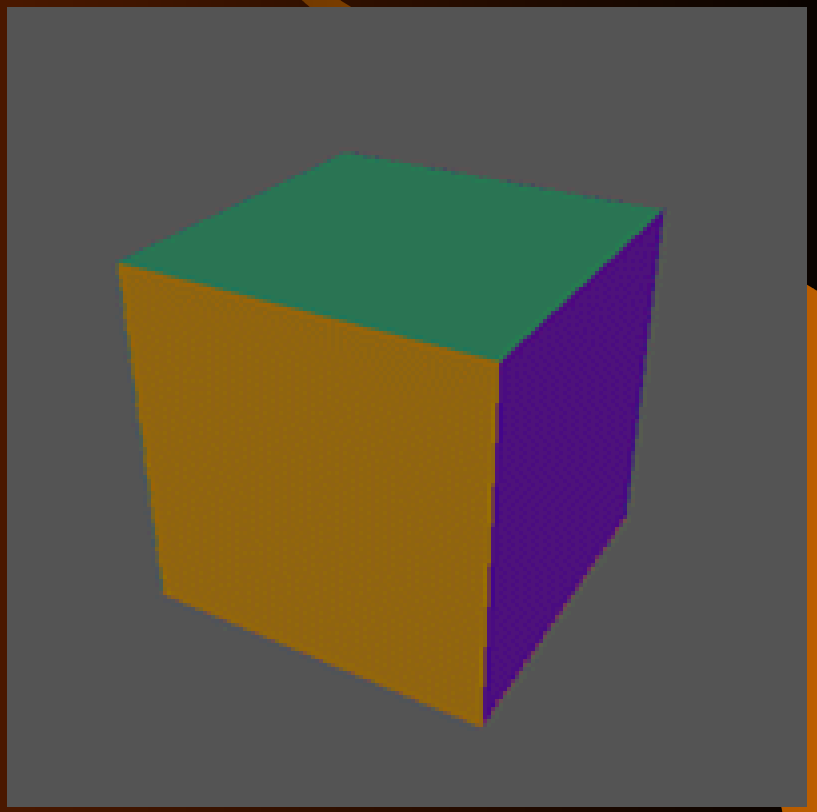
torus

Flat torus



Flat 3-torus

- Glue:
  - Top to bottom
  - Left to right
  - Front to back



Flat 3-torus

- Have a seat in a flat 3-torus.
  - What do you see when you look
    - up?
    - down?
    - left?
    - right?
    - straight ahead?
    - backwards?

Question

- A town is built in a flat 3-torus.
  - All North-South streets are 1-way northbound
  - All West-East streets are 1-way eastbound
  - Elevators only go up.
- Can you go from any place to any other place in this town?

Characteristics of a flat 3-torus

- How many dimensions does it have?
  - 3
- Is it finite or infinite?
  - finite
- Does it have boundary?
  - no

Goal

Flat 2-torus and flat 3-torus are both examples of “nice” spaces

(called *manifolds*)

Goal: **define “nice”**

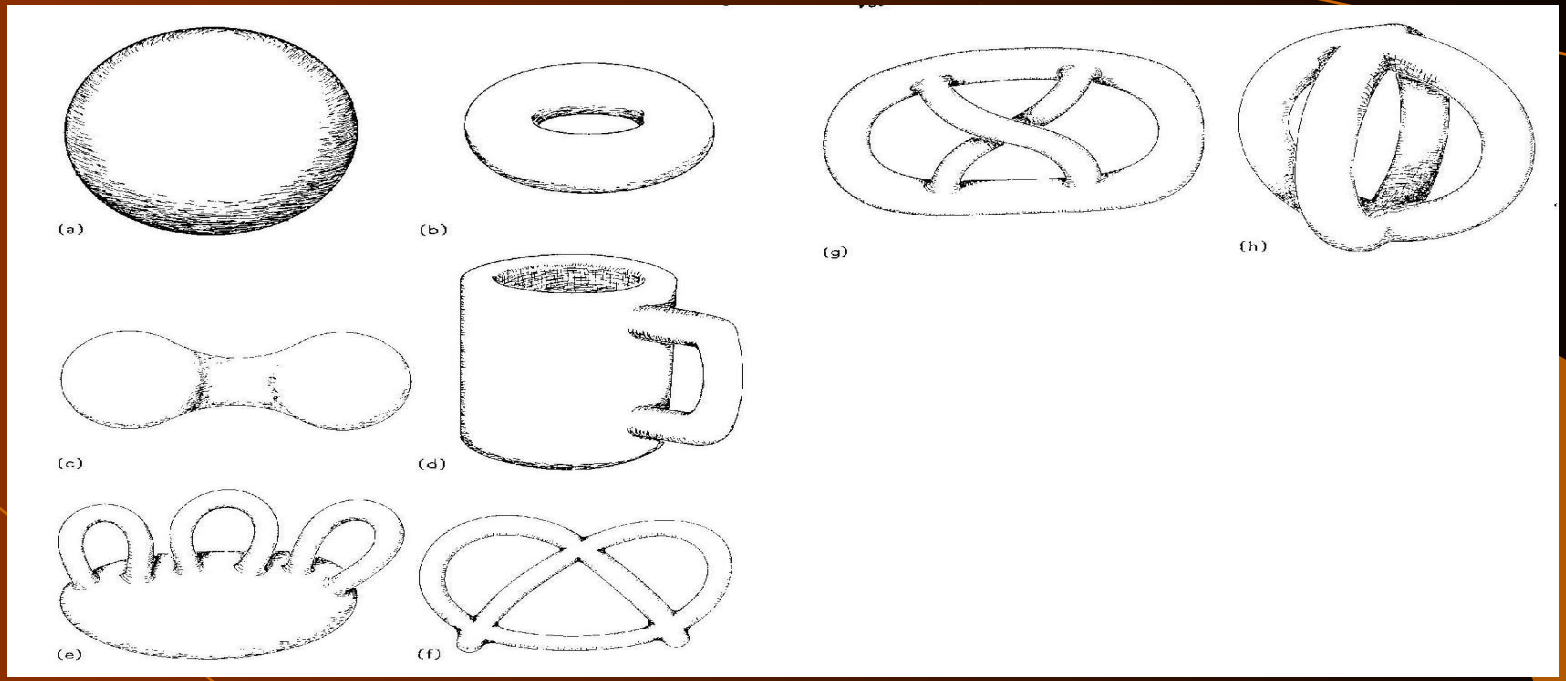
Allowed moves

- Bend! Stretch! Twist! Flip and flop!
  - DO NOT TEAR
  - DO NOT GLUE

Topology vs. Geometry

- The aspects of a space that remain unchanged under this deformations are called space's TOPOLOGY
- The aspects of the space that do change when you deform the space are called geometry (distances, angles, areas, curvature…)

Exercise 1



- Which spaces have the same topology?
  - a & c; b &d; e & f & g & h

Exercise 2

cylinder

Cut the cylinder open, put a full twist in one of the ends and glue it back up.

Does the new space have the same topology?

Exercise 2 cont'd

- The topology changed from our (outsider) point of view!
- But what if we were the insiders? Did the topology change from a point of view of somebody from within the surface?
  - No, it did not change.

Exercise 2 cont'd

- The INTRINSIC topology of the surface did not change, but its EXTRINSIC topology (the way it sits in a 3-dim'l space) did.

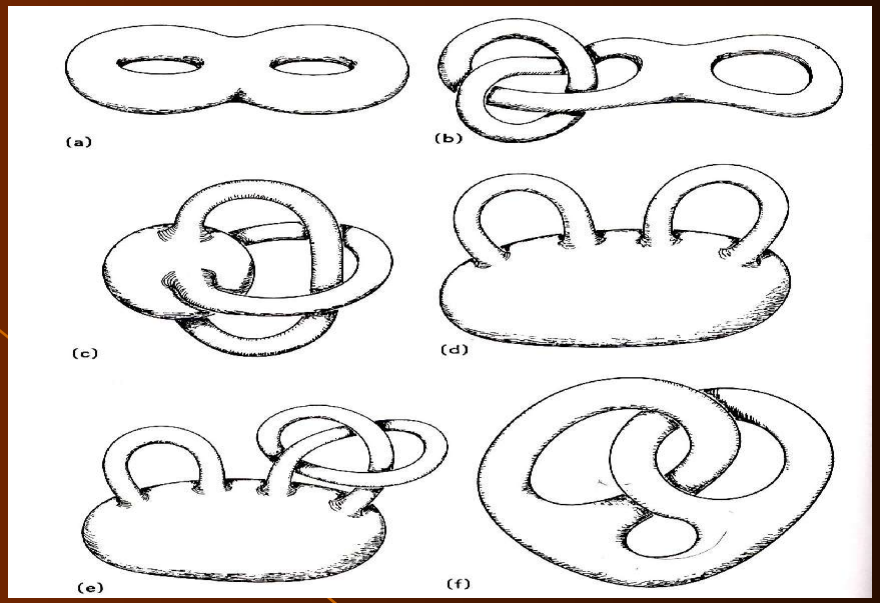
Extrinsic vs. Intrinsic Topology

- Two spaces have the same *intrinsic topology* if they can not be told apart from within.

- Two spaces have the same *extrinsic topology* if one can be deformed (inside a larger space) to look like the other.

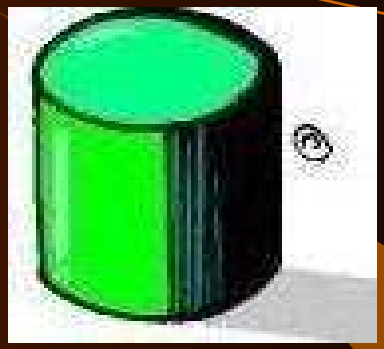
Exercise 3

- Convince yourselves that all the spaces have the same intrinsic topology. Which ones have the same extrinsic topology as well?
  - a & c & d & f; b & e



Exercise 4

Flat cylinder



cylinder

Do these two spaces have same or different:

- intrinsic topology? • same
- extrinsic topology? • different
- intrinsic geometry? • same
- extrinsic geometry? • different

Geodesics

- Intrinsically straight lines are called geodesics
- To find geodesics you must live inside the space and you should pull a cord taught between two points

Gauss' experiment

- Gauss tried to measure the curvature of the Universe by measuring the angles in a triangle formed by three mountain peaks.

Question

- Can you decide if the Earth has a whole by looking into your back yard?
  - Local vs. Global Properties
  - Homogeneous vs. Nonhomogeneous spaces

Local vs. Global Properties

- Local properties are those observable on the small region of your space.
- Global properties can only be observed by considering the whole space.

Exercise 5

- Do torus and flat torus have the same:
  - Intrinsic global topology
  - Extrinsic local geometry?

- yes
- no