

# New notions

Dimension, finiteness and other fun  
stuff

# Question

- How would you specify a position of a train traveling from Montreal to Toronto?
- How would you specify a location of a point on a line?

# 1 dimension

- If you need only one number to specify a position of a point in your space, then your space is 1-DIMENSIONAL.
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- Line is a 1-dimensional Euclidean space,  $E^1$ .

# Question

- How can you specify a position of a sailboat on an ocean?
- How many numbers are needed to specify a position of a point on a piece of paper?

# 2 dimensions

- If you need two numbers to specify location of a point in your space, then that space is 2-dimensional.



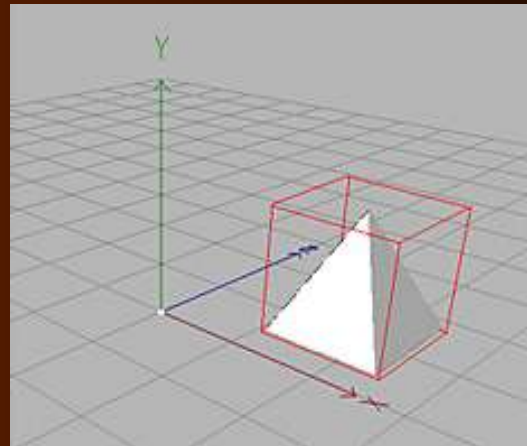
- Plane is a 2-dimensional Euclidean space,  $E^2$ .

# Question

- How would you specify a position of an airplane in Earth's atmosphere?
- How many numbers are needed to describe a point inside this room?

# 3 dimensions

- If three numbers are needed to describe a position of a point, then the space in question is 3-dimensional.



- Space, as we perceive it, is a 3-dimensional Euclidean space,  $E^3$ .

# Exercise

- What is the dimension of each of the following spaces?
  - circle
  - surface of the Moon
  - inside of the Moon
  - blackboard
  - milk in the milk carton
  - surface of your skin
  - time on a given day



# Food for thought

- What would  $E^n$  be?

A Euclidean space in which we need  $n$  numbers to describe a position of a point.

- Can you see 4-dimensional space?
- Can a 2-dimensional space have volume?

# Time for art

- Draw a 2-dimensional person.
  - How do they see?
  - How do they eat?
  - Where is their skin?
  - How do they eat and digest food?

# More...

- Draw a 2-dimensional planet.
- Where on it do Flatlanders live?
- Are the ones you drew earlier possible inhabitants of this planet?

# 1-dimensional planet

- Make a 1-dimensional planet with few linelanders.
- Where is their skin? Their eyes?
- Can they change neighbors?

# Finite vs. Infinite

- Space is FINITE if it has measurable length/area/volume/....
- Space is INFINITE if has unlimited length/area/volume/...

# Boundary

- Boundary = Edge
- Once you hit boundary you can not go past it, you have to turn around.

# In dimension 1

	finite	infinite
With boundary		
Without boundary		

# In dimension 2

	finite	infinite
With boundary		
Without boundary		



# In dimension 3

	finite	infinite
With boundary		
Without boundary		

# Story of Flatland

- Battle:
  - Plane vs. Sphere
- Imagine this:
  - You get on you little space ship, travel to a distant Galaxy, head towards the friendliest looking planet. It's EARTH!!!!