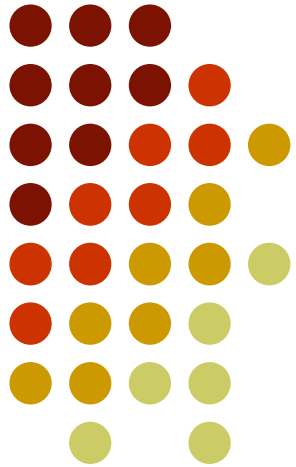
Fractions II

Comparing, adding, subtracting

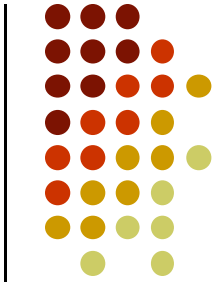


Questions

- Are whole numbers fractions?

- If Annie got 1/8 of a pie and Manny got 1/13 of a pie, who ate more pie?
- If Annie got 2/3 of a pie and Manny got 3/4 of a pie, who ate more pie?
- How can we compare fractions? Are there some that are easier to compare?

Comparing fractions



- Convert fractions into equivalent ones that are easier to compare.
- OR
- Use your fraction sense.

$$\frac{2}{3} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$$

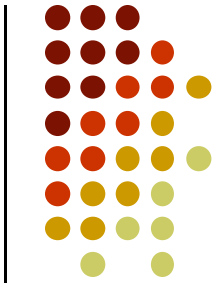
$$\frac{2}{3} < \frac{3}{4}$$

But, I could also think about how far each of these is from 1!

Problem

- Arrange these fractions from smallest to largest without converting to equivalent fractions, decimals, drawing pictures. Use your fraction sense and reasoning tools:

$\frac{3}{4}$ $\frac{2}{5}$ $\frac{5}{6}$



- Is it possible to put a fraction between any two fractions on a number line?
- Why or why not?
- We say that the set of fractions is dense.

Addition of fractions

- Can you come up with a way of adding two fractions with equal denominators? Give an example and show on a model why your method works.

- What about adding fractions with different denominators?

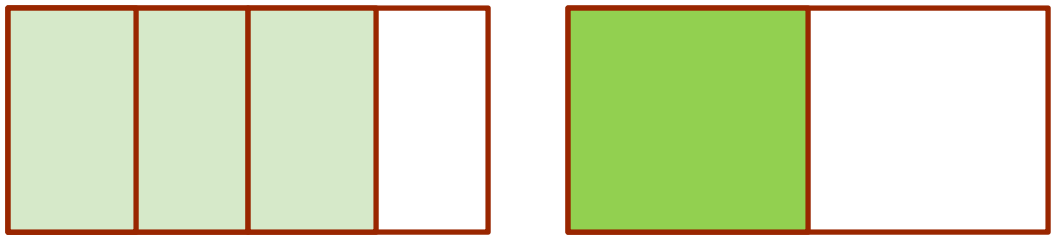
Definition

- Let $\frac{a}{b}$ and $\frac{c}{d}$ be any two fractions. Then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Misconceptions

- Some students initially view the addition of fractions as adding the numerators and denominators as follows: $\frac{3}{4}+\frac{1}{2}=\frac{4}{6}$



Using this example discuss why such method for addition is unreasonable.

Exercises:

$$\frac{1}{8}+\frac{5}{8}=$$

$$\frac{3}{7}+\frac{1}{3}=$$

$$\frac{8}{9}+\frac{1}{12}+\frac{3}{16}=$$

$$2\frac{2}{3}+1\frac{1}{4}=$$

$$\frac{2}{3}-\frac{1}{4}=$$

$$\frac{13}{18}-\frac{8}{27}=$$

Properties?

- What properties of whole number addition do you think fraction addition has?
  - Closure?
  - Commutativity?
  - Associativity?
  - Additive identity?

Subtraction

$$\frac{5}{8} - \frac{1}{8} =$$

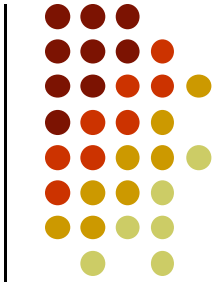
$$\frac{2}{3} - \frac{1}{4} =$$

Definition

- Let $\frac{a}{b} \geq \frac{c}{d}$ be any two fractions. Then

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

Mental math and properties of addition



$$\left(\frac{3}{7}+\frac{1}{9}\right)+\frac{4}{7}=$$

$$\left(2\frac{2}{5}+3\frac{5}{8}\right)+\left(1\frac{4}{5}+2\frac{3}{8}\right)=$$

$$8\frac{2}{7}-2\frac{6}{7}=$$

$$4-2\frac{3}{9}=$$

Multiplication: a whole number times a fraction

- Use the repeated addition approach:

$$3 \times \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

Multiplication: a fraction times a whole number

- Repeated addition doesn't make sense for

$$\frac{1}{3} \times 6$$

- We can think of taking one third of 6.

Multiplication: a fraction times a fraction

- Use the new approach!

$\frac{1}{5} \times \frac{3}{4}$ is one fifth of $\frac{3}{4}$

Picture $\frac{3}{4}$

Take one fifth:

$$\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$$

Definition

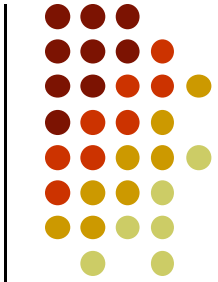
- For any two fractions $\frac{a}{b}, \frac{c}{d}$ we define

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Question

- Is $2\frac{1}{3} \cdot 3\frac{1}{2} = 6\frac{1}{6}$? Why or why not?

Properties of multiplication



- All the properties of whole number multiplication and
- Multiplicative inverse property:
  - For every nonzero fraction $\frac{a}{b}$ there is a unique fraction $\frac{b}{a}$ such that

$$\frac{a}{b}\cdot\frac{b}{a}=1$$

Problem

- During one evening Kathleen devoted 2/5 of her time to mathematics, 3/20 of her time to Spanish, 1/3 of her time to biology and the remaining 35 minutes to English. How much time did she spend studying her Spanish?