Sets, …

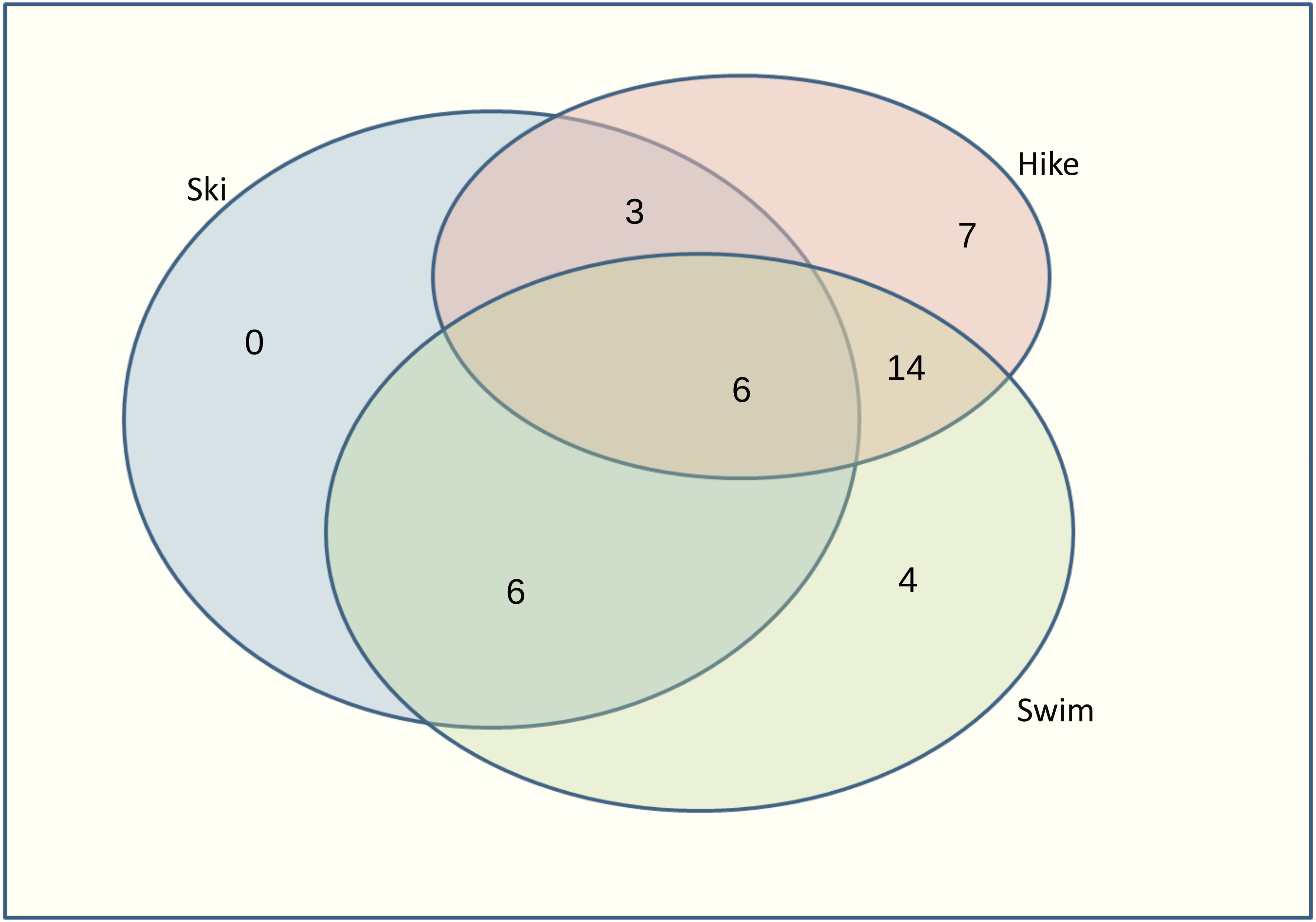
Toward numbers

Poll – I love to …

- Ski
- Hike
- Swim
- Ski and hike
- Ski and swim
- Hike and swim
- All three

How many people:

- like to swim but not to hike or ski?
- like to do exactly one of the three?
- like to hike and swim, but not to ski?



People in this room

Venn diagram

A collection of objects is called a *set.* The objects that belong to the set are called *members* or *elements* of the set.

Ski={all people in this room who like to ski}
={P: P is in this room and likes to ski}
Swim={all people in this room who like to swim}
Hike={all people in this room who like to hike}

U={all people in this room} -- *universe*

Question

- What can you tell me about these two sets:
  - A = {set of all people in this room whose name is not Emina}
  - B = {set of all students in this room}?

A=B

Two sets are *equal* if they have exactly same elements.

- Emina does not belong to set B, and we say that Emina is not an element of B:
  - Emina $\notin$ B

- Is Joe an element of B?
  - Joe $\in$ B.

Question 2

- M={Brittney, Julie, Robert, Sarah}

- How does M compare to A?

$M \subseteq A$

M is a subset of A

Set X is a *subset* of set Y if and only if every element of X is also an element of Y. We write $X \subseteq Y$.

- Since A contains elements that are not in M, we can also say that M is a *proper subset* of A:

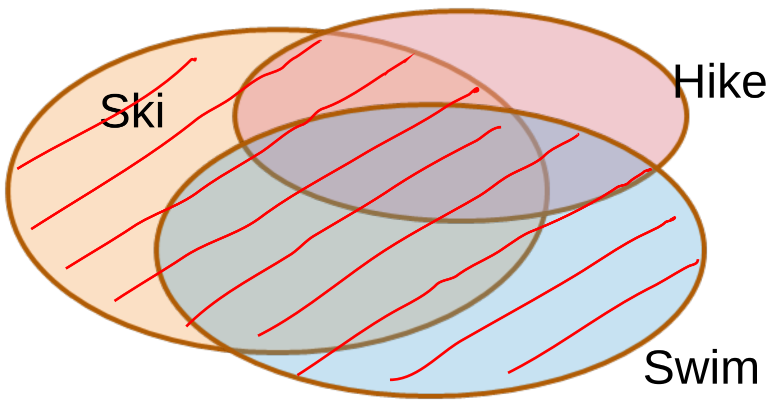
$$M \subset A$$

- If two sets have no elements in common they are called *disjoint.*
- Can you name two disjoint sets?

Ski={all people in this room who like to ski}
Swim={all people in this room who like to swim}
Hike={all people in this room who like to hike}

U={all people in this room} -- *universe*

Ski ∪ Swim = {all people in this room who like to ski or swim}



The *union* of two sets X and Y, $X \cup Y$, is the set of all elements that belong to X or Y.

$$X \cup Y = \{x : x \in X \text{ or } x \in Y\}$$

Note:

$$X \subseteq X \cup Y$$
$$Y \subseteq X \cup Y$$

Examples

- Remember B = {set of all students in this room}
- What is

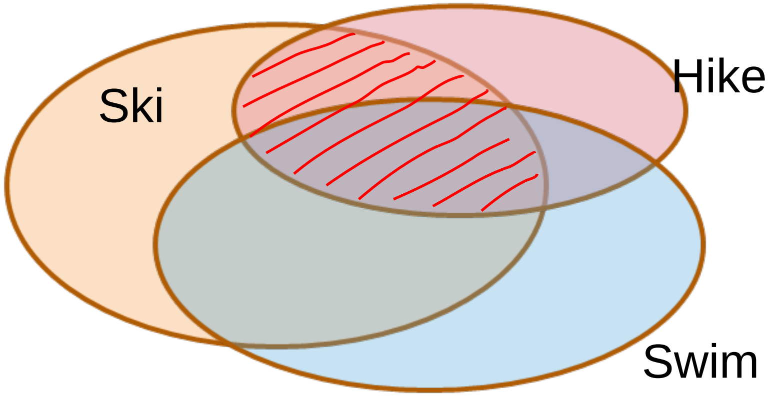
{Emina} ∪ B = **U** = {all people in this room}={Emina and the students in this room}

Swim ∪ **Hike** = {all people in this room who like to hike or swim}

{t, n} ∪ {l, t, n} ={t, n, l}

U ∪ **Swim** = **U**

Ski ∩ **Hike** = {all people in this room who like to ski and hike}



The *intersection* of two sets X and Y, $X \cap Y$, is the set of all elements that belong to both X and Y.

$$X \cap Y = \{x: x \in X \text{ and } x \in Y\}$$

Note:

$$X \cap Y \subseteq X$$
$$X \cap Y \subseteq Y$$

Exercise

- What is

{Emina} $\cap$ **Swim** = $\emptyset$

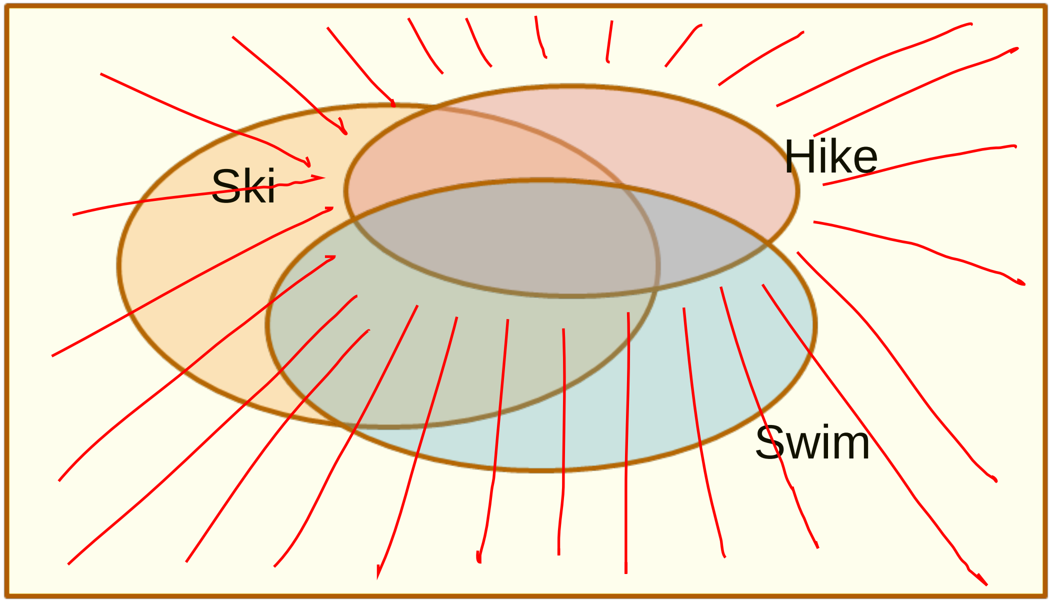
{a, x, d} $\cap$ {p, d, f} = {d}

U $\cap$ **Ski** = **Ski**

Empty set is a set that contains no elements, $\emptyset$.

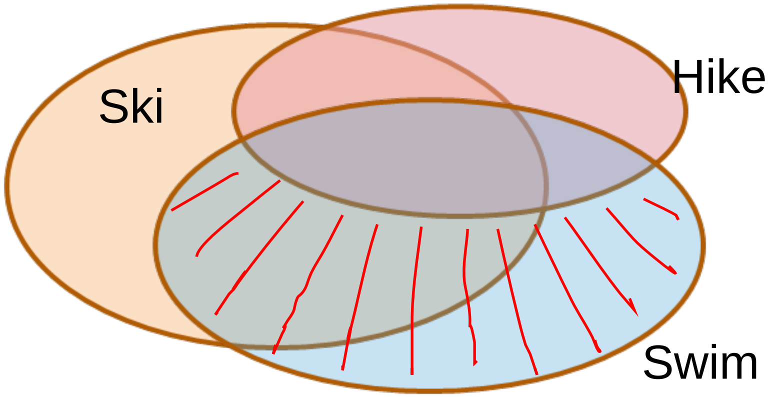
Is {$\emptyset$} an empty set? No, it is a set whose element is an empty set.

$\overline{\textbf{Hike}}$ = {all people in this room who do not like to hike}



The *complement* of set X is the set of all elements in the universe that do not belong to X.

Swim \ Hike = {all people in this room who like to swim but do not like to hike}



The *difference* of two sets X and Y, $X \setminus Y$, is the set of all elements that belong to X but do not belong to Y.

$$X \setminus Y = \{x: x \in X \text{ and } x \notin Y\}$$

Note:

$$X \setminus Y \subseteq X$$

Problem

- If U={1, 2, 3, 4, 5, 6}, A={2, 3, 5} and B={1,4} verify in two different ways that

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$A \cup B$ ={1, 2, 3, 4, 5}

$\overline{A \cup B}$ = U \ (A ∪ B) = {6}

$\overline{A}$ ={1, 4, 6}

$\overline{B}$={2, 3, 5, 6}

$\overline{A} \cap \overline{B}$ ={6}

We see that both sets contain only one element, and that element is 6, hence the Two sets are equal.

Question

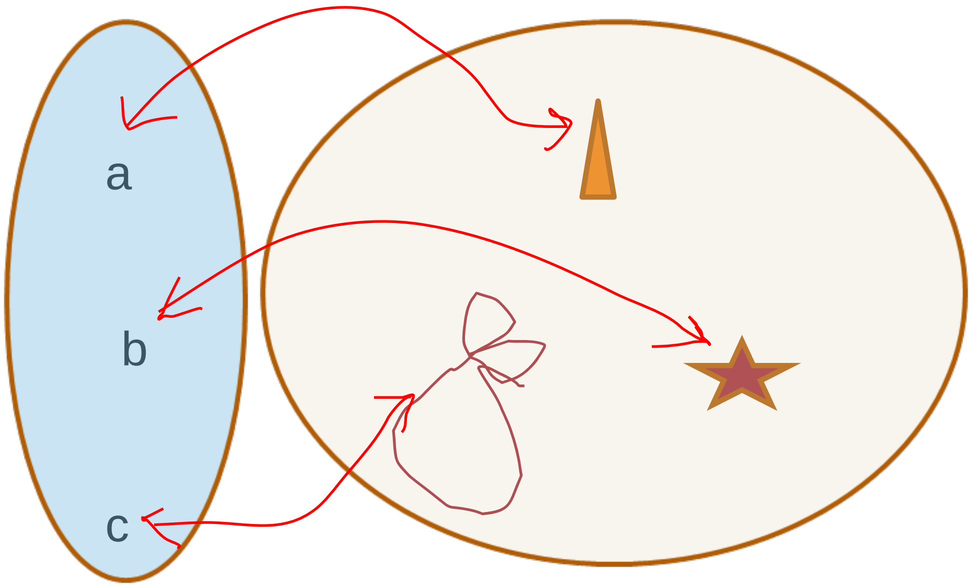
- A={all people in this room}

noses

- B={all ~~bags~~ in this room}

How could we pair the elements of these two sets?

- A *1-1 correspondence* between two sets A and B is a pairing of their elements so that to every element of A corresponds exactly one element of B, and vice versa. If there is 1-1 correspondence between A and B we say that A and B are equivalent, A ~ B.



Questions?

- Are equivalent sets necessarily equal?
- Why or why not?

Exercise

- Give me 3 different 1-1 correspondences between sets A={a, b, c, d} and B={♣,♦,♥,♠}

- Is there a 1-1 correspondence between sets X={x, y, z} and Y={a, b}?

what about z?

A strange hotel

- There was a hotel whose rooms were numbered 1, 2, 3, 4, 5, … and in each room there were two people. After the first night each pair decided that they'd much rather have a room for themselves. They talked to the manager and he said "Not a problem at all!" How is this possible?

- Hint:

Can you set up a 1-1 correspondence between sets

N={1, 2, 3, 4, 5, 6, 7, … }

E={2, 4, 6, 8, 10, … }