M4010 Midterm #1 Fall 2007

Name: Key Show work! All work!

1₃. Short answers: *(16 pts)*

a. The cube (or block) represents what in base 4? $4^3 = 64$

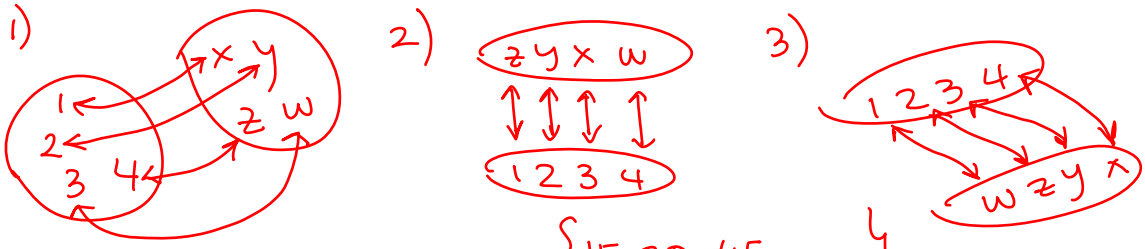
b. Your favorite problem solving strategy is draw a diagram

c. A set B is a subset of set C if every element of B is also an element of C.

d. The set of digits needed for base 4 numbers = {0, 1, 2, 3}

e. 10110_2 = $1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2 = 16 + 4 + 2 = 22$ in base ten.

f. Show three different correspondences between sets {1, 2, 3, 4} and {x, y, z, w}



g. { 5, 10, 15, …} ∩ { 3, 6, 9, 12, ….} = {15, 30, 45, ...}

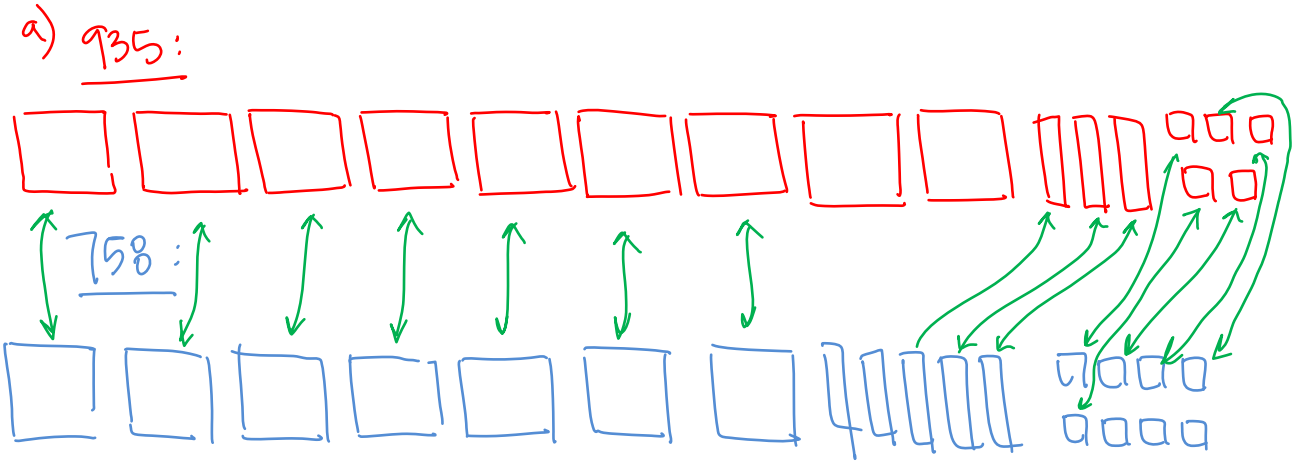
h. { e,a,s,y} U {s,e,a} = {e, a, s, y}

2₃. Do two of the following three. Cross through the one you do not wish to be graded *(8 pts)*

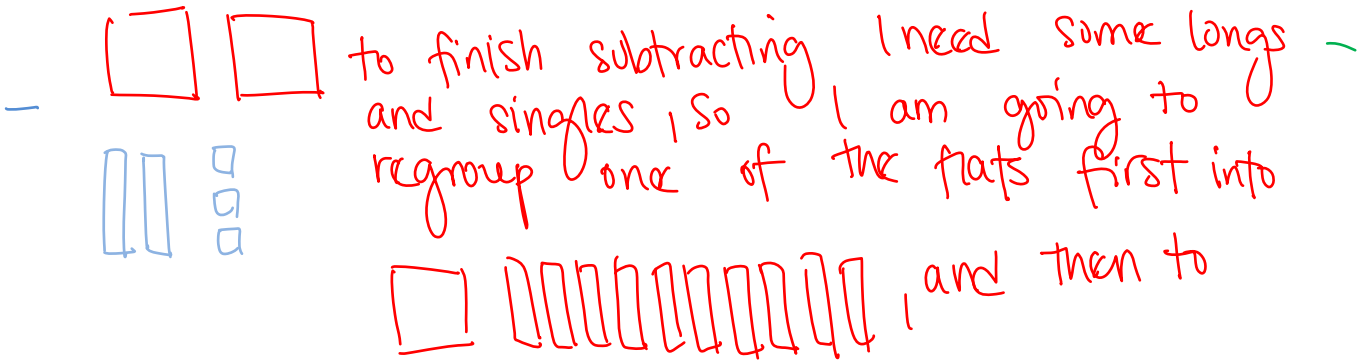
a. 935 – 758 using base ten blocks clearly marking all the trading and regrouping you do.

b. $23_6 - 5_6$ using a number line.

c. $231_5 + 423_5$ using base five blocks.

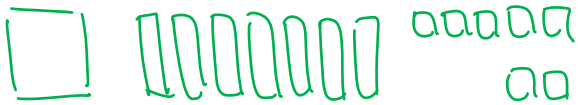


This can be done in several ways. I'll start matching up pieces from 758 with corresponding pieces in 935, which will be the ones we take away and we'll keep track of what is left to do. I'll start in green in the above picture. So what we have left is



to finish subtracting I need some longs and singles, so I am going to regroup one of the flats first into

, and then to



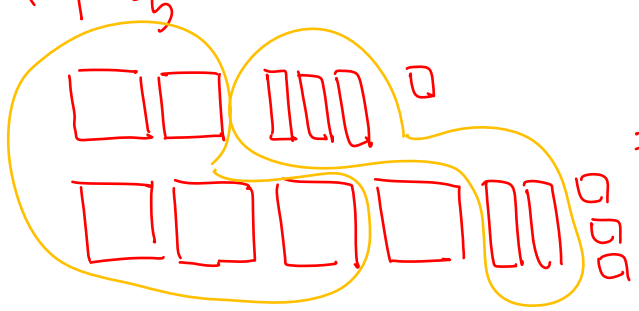
So $935 - 758 = 177$

b) $23_6 - 5_6 = 14_6$

c) $231_5 + 423_5$

231_5 :

423_5 :



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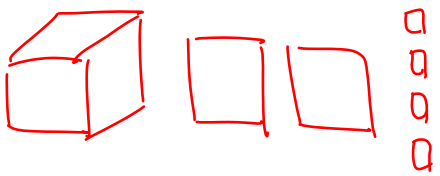


$1204_{(5)}$

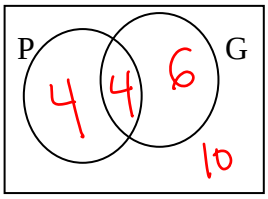
10_3 . Complete and shade this Venn diagram: *(15 pts)*

Fill in all numbers in this diagram. 24 students were interviewed:

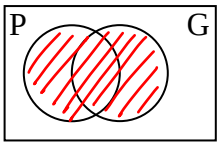
8 said they play the piano.

10 said they play the guitar.

6 said they only play the guitar.



a.

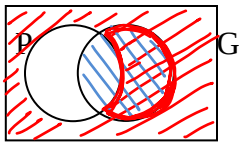


Shade $P \cup G$

Describe that person in words:

The students who played a guitar or piano.

b.



Shade $\overline{P} \cap G$

Describe that person in words:

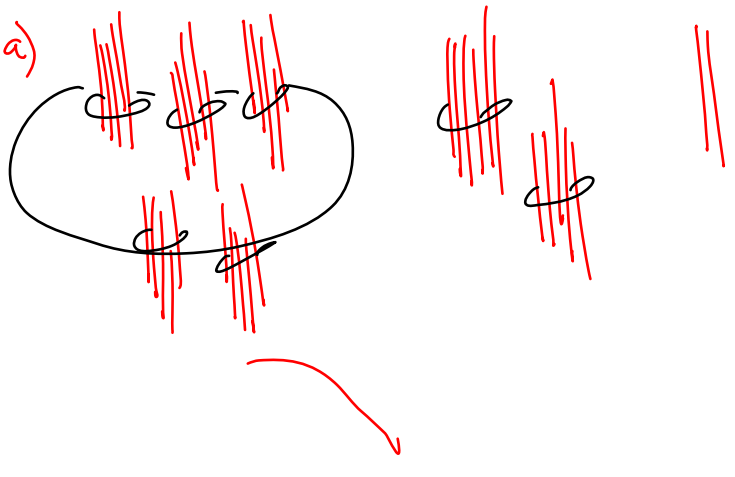
Students who played a guitar but did not play piano.

11_3 . Do three of the four on this problem. Cross out the one you are not doing.

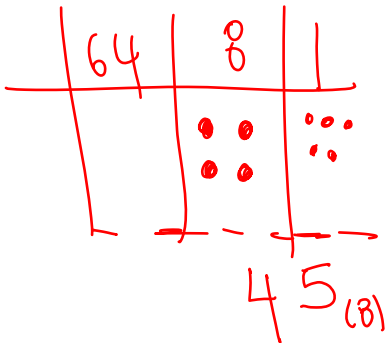
(12 pts) Write the base ten number thirty-seven in **three of the four** ways (clearly mark which one you are working out):

a. Bundling sticks with bundles of 5
b. Chip abacus with base 8
c. flats, longs and units, base three
d. numerals in base 2

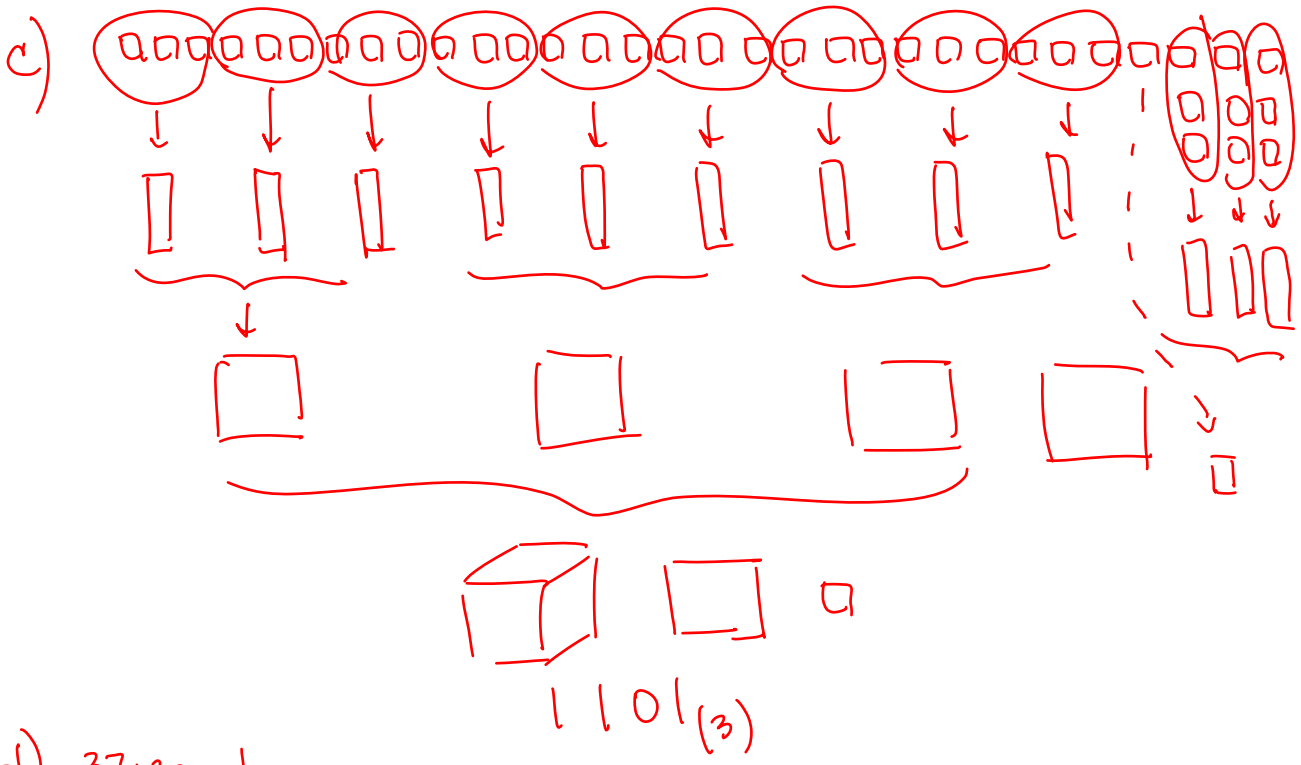
a)



b)



c)



$1101_{(3)}$

d) $37 : 32 = 1$ R 5

$5 : 4 = 1$ R 1

32	16	8	4	2	1
1	0	0	1	0	1

$100101_{(2)}$

12₃. *(24 pts)*

a. Write all of the subsets of {I, ♡, M}. How do you know you have all them?

$\emptyset$, {I}, {♡}, {M}

{I, ♡} {♡, M}

{I, M} {I, ♡, M}

A set of 3 elements has $2^3 = 8$ subsets. In general, a set with n elements has 2^n subset

One way to see this is to notice that a subset can have at most n elements. Let's say $A = \{1, 2, 3, \ldots, n\}$ and we need to know how many subsets A has.

If B is a subset of A then I can think of building B as follows $B = \{\_, \_, \_, \_, \ldots, \_, \_\}$

I'll put n empty slots and each will represent an element of A, in the same order in which they appear in A. Then for each slot I have 2 choices: either

I will put that element in B or I will not. So we have $2 \cdot 2 \cdot 2 \cdots \cdots 2 = 2^n$

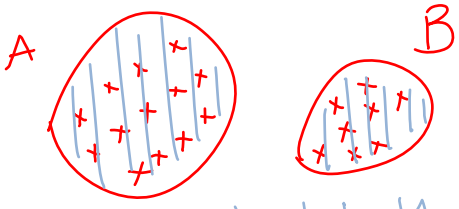
two choices for first el. 2 choices for second el. 2 choices for third el. 2 choices for n^{th} el.

possible subsets.

b) If set A has 13 elements and set B has 8, what are the greatest and the least number of elements which could be in each of these. Make **drawings** to support your answer.

$A \cup B$: greatest $13+8=21$

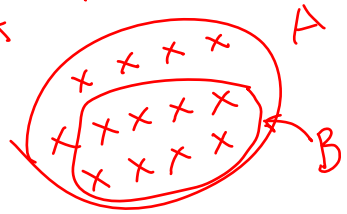
this happens when A and B are disjoint



AUB is shaded blue, and has 21 elements.

least 13

This happens when $B \subseteq A$

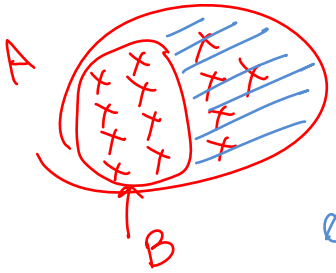


AUB = A and has 13 elements.

A-B greatest 13 and this happens when A and B are disjoint

A-B is shaded blue and has 13 elements

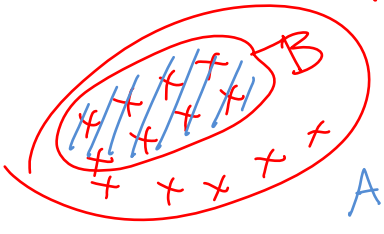
least $13-8=5$ and this happens when $B \subseteq A$



A-B has 5 elements

$A \cap B$ greatest 8

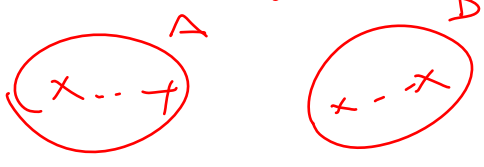
when $B \subseteq A$ then $A \cap B = B$ and has 8 elements



$A \cap B = B$

least 0

when $A \cap B = \emptyset$ that is when A and B are disjoint



20_3. <u>Do 2 of these 3 and cross clearly the one you do not want to be graded</u>. *(24 pts)*

a) Kisha is 12 years old and her brother Albert is 2 years old. In how many years will Kisha be twice as old as Albert?
b) Fierrante rents a compact car for $25 a day plus 20 cents per mile. If he rents the car for one day, how far can he go for $50?
c) A frog sits at the bottom of a 15 foot deep well. Each day he climbs 3 feet, and each night he falls back 2 feet. How long will it take the frog to get out of the well?

a) In y years from now Kisha will be $12+y$ years old and her brother will be $2+y$ years old. We would like to know for which y we will have

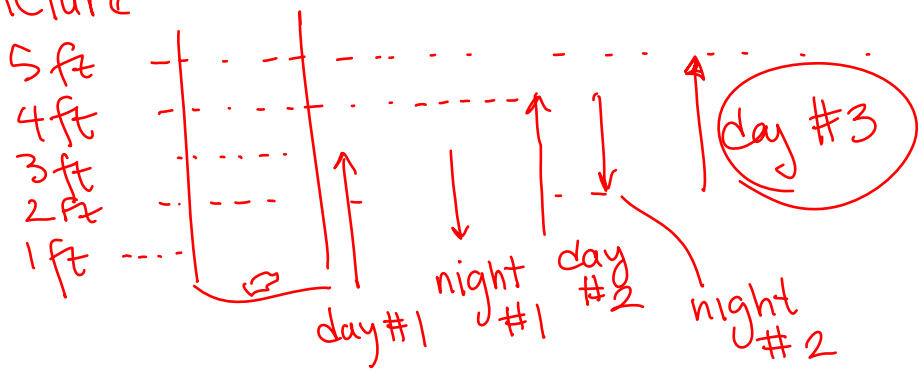
$$(12+y) = 2(2+y)$$
$$12+y = 4+2y$$
$$8 = y$$

In 8 years Kisha will be 20 and Albert will be 10, which is half of Kisha's age.

b) If Fierrante rents a car for one day then he'll have to pay daily fee of $25 which will leave him with $25 to spend on mileage. Each mile costs him 20 cents = $0.2 so he can drive

$$\frac{\$25}{\$0.2/\text{mile}} = 125 \text{ mi}$$

c) If the well is 5 feet deep we can draw a little picture

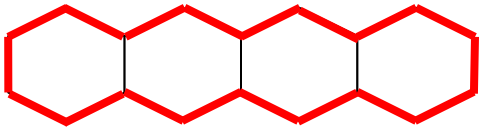


so it took the frog 3 days to climb out of 5 foot well.

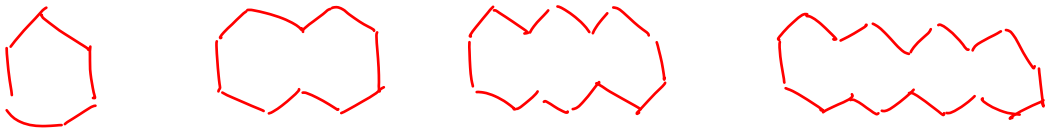
We also notice that after one day & one night the frog has advanced 1 foot except the last day. The last day the frog can go 3 feet & get out of the well. So, if the well is 15ft deep then once the frog is at 12 feet it just needs one day to get out. But for those first 12 feet it will need a day for each foot, so we have a total of 13 days.

21₃. <u>Do one of these two problems</u>. Cross out the one you decide not to do.*(14 pts)*

a) A "hexagon train" is formed by placing hexagons side-by-side, in a row. The perimeter of a hexagon train equals the number of units around the outside of the entire train. For example, the perimeter of this hexagon train is shown in bold:

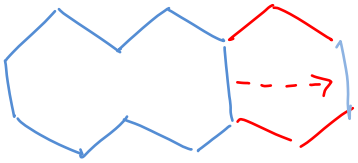


a) What is the perimeter of the hexagon train made with 4 hexagons? 5 hexagons? 100 hexagons?
b) Find a way to express the perimeter of a hexagon train made with any number of hexagons. Write a clear and detailed explanation of your method.
c) Is there a hexagon train with a perimeter of 583 units? Why or why not?



6 10 14 18

It appears that every time we add a hexagon the perimeter increases by 4. This can be explained on the picture b/c adding a new hexagon (say on the right) amounts to adding a roof ^ and an upside down roof v while the vertical 'side' is just moved over to the right. For example from figure 2 (in blue) to figure 3:



So the first figure has 6 sides and each new is 4 bigger than the previous one.

fig #	1	2	3	4	5	6	...	100
per.	6	10	14	18	22	24	...	?

$P_2 = P_1 + 4$

$P_3 = P_2 + 4 = P_1 + 2 \cdot 4$

$P_4 = P_3 + 4 = P_1 + 3 \cdot 4$

$\vdots$

$P_n = P_{n-1} + 4 = P_1 + (n-1) \cdot 4$ ←

$P_{100} = P_1 + 99 \cdot 4 = 402$

Instead of representing P_n in terms of P_{n-1} we can trace it back all the way to P_1 and have an expression in terms of P_1 and the figure's number:

$P_n = P_{n-1} + 4 = P_{n-2} + 4 + 4 = P_{n-2} + 2 \cdot 4 = P_{n-3} + 4 + 2 \cdot 4 = P_{n-3} + 3 \cdot 4 =$

$= \dots = P_1 + (n-1) \cdot 4 = 6 + (n-1) \cdot 4$

c) The question becomes: Is there an n such that $P_n = 583$? If there were then we would have

$$6 + (n-1)\,4 = 583$$

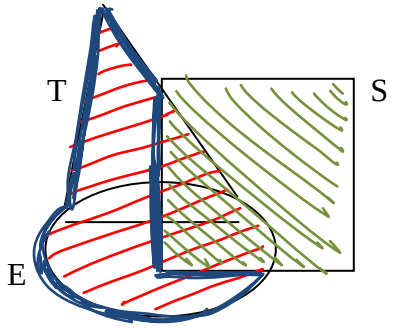
$$4(n-1) = 577$$

and we can see that there is no whole number n that would work here (577 is not divisible by 4).

So there is no train that is 583 long.

b. Shade these and describe in words who is in the set.

T = Trumpet players S = Sophomores E = Excels in math



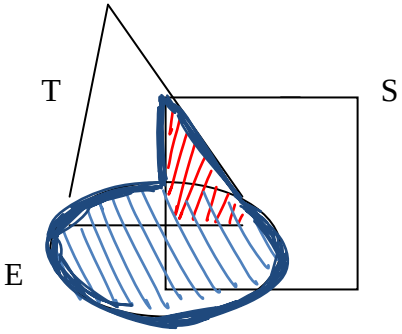
Shade $(T \cup E) - S$ and
Describe that person in words:

TUE ///

S \\\

(TUE) − S will be the red only. I am outlining the set in blue.

* A person who is in (TUE) − S is a student who plays a trumpet or excels in math, and is not a sophomore.



Shade $(T \cap S) \cup E$ and
Describe that person in words:

T∩S ///

E \\\

(T∩S)UE is the set that is shaded either red or blue or both.

* A person in this set is a student who excels in math or a sophomore who plays a trumpet.