Creating the Maximum box

Assignment: You will be given a rectangular sheet of paper: Your goal is to make a topless box of the greatest possible volume (one which holds the most.)

a. Decide what size square you will cut from the corners of the paper.
b. Cut a square from each side. (All squares the same size.)
c. Fold up the sides of the box and tape them.

Inside the box you will write the following information:
(This will be easier to do if you write it before taping your box.)

1) The dimensions of the original rectangle of paper (cm.)
2) The size of the square you cut from the corners (cm.)
3) The dimensions of your box (l, w, h).
4) The volume of the box (l*w*h).
5) The surface area of the box.
6) How you determined the size of square to cut from the corners.
7) Your name and period, of course!

Dress the box up if you like!

Phase 2:

1. Plot the class data on a graph:
   x = the size of square cut out
   y = the volume of the box.

   a. What values of x made it impossible to make a box?
   b. What is the maximum volume you were able to get and what value of x produced it?
   c. Is it possible to have two different boxes which have the same volume?

2. Try to write an equation which hits as many of the points as possible.

3. Now let's do the algebra. The volume formula for a box is V= l*w*h. Can we write the length, width and height in terms of x?

4. Graph the equation on the same screen as your plot.

Here is a problem from a text:

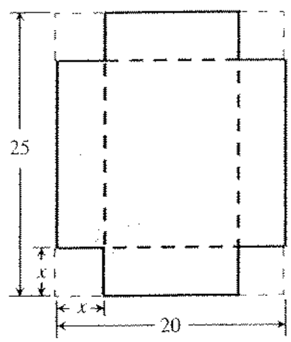


Figure 3.10 Squares cut from a piece of cardboard measuring 20 by 25 in.

EXAMPLE 5 APPLICATION: Finding the Maximum Volume

Squares are cut from the corners of a 20- by 25-in. piece of cardboard, and a box is made by folding up the flaps (see Fig. 3.10). Determine the graph of the problem situation and find the dimensions of the squares so that the resulting box has the maximum possible volume.

Solution

x = width of square and height of the resulting box in inches

$20 - 2x$ = width of the base in inches

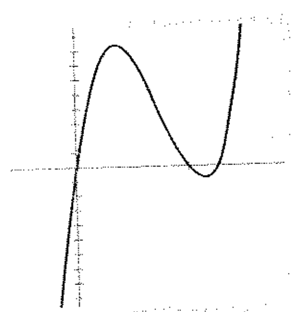
$25 - 2x$ = length of the base in inches

$V(x) = x(20 - 2x)(25 - 2x)$ Volume is equal to length × width × height.

Find a complete graph of $V(x) = x(20 - 2x)(25 - 2x)$ (see Fig. 3.11a). In this problem situation, $0 < x < 10$, so the complete graph of the problem situation is shown in Fig. 3.11(b).

Use zoom-in to find the coordinates of the local maximum (see Fig. 3.11c). Read the coordinates of the high point as (3.68, 820.5282) with an error of at most 0.01. Thus the maximum volume is 820.5282 cu in. and this volume occurs when $x = 3.68$ in.

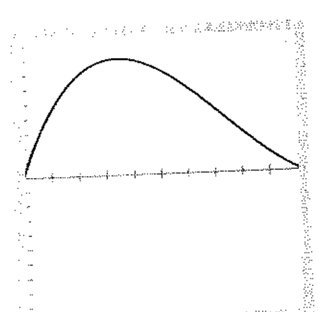
Graph of the algebraic equation:



[−5, 20] by [−1000, 1000]

(a)

Graph of the problem situation:



[0, 10] by [−1000, 1000]

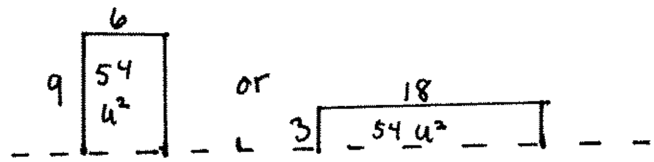
(b)

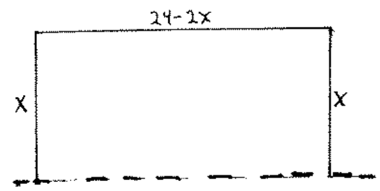
Why must a pig pen be a rectangle?

Given 24 feet of fence to build a pen for your pigs using a river as one boundary, what are the possible shapes it may take and which produces the most area for the 24 feet of fence?

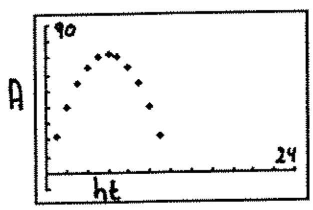
1. Examine the possible11 rectangles: A = (b)(h)

Ht	Width	Area
1	22	22
2	20	40
3	18	54
4	16	64
5	14	70
6	12	72
7	10	70
8	8	64
9	6	54
10	4	40
11	2	22

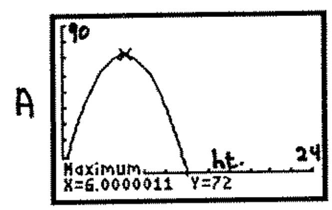




Plot these values:

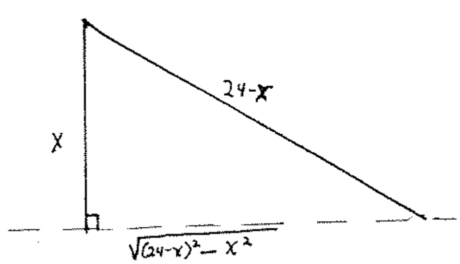


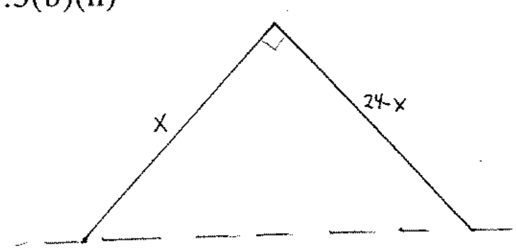
Graph the function: A(x) = (24-2x)(x)



What if we do not want a rectangle? Let's examine a few other shapes:

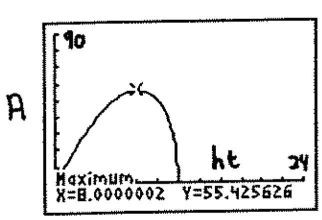
2. Examine some right triangles: A = .5(b)(h)





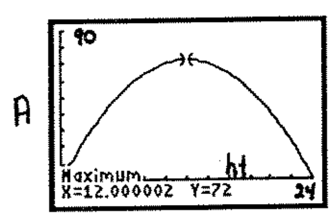
leg on the river

$A(x) = (.5)(x)(\sqrt{(24-x)^2 - x^2}$



hypotenuse on river

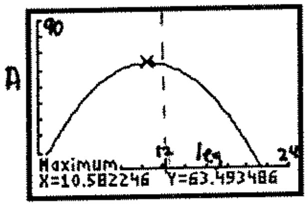
A(x) = (.5)(x)(24-x)



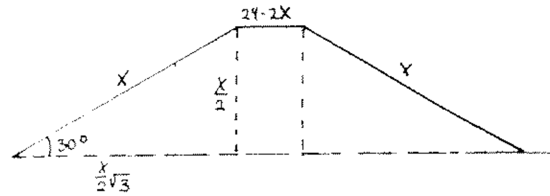
3. Trapezoids: $A = .5(h)(b_1 + b_2)$

Isosceles trapezoid with 30° base angles.

$$A(x) = (.5)(\frac{x}{2})(2(24-2x)+2(\frac{x\sqrt{3}}{2}))$$

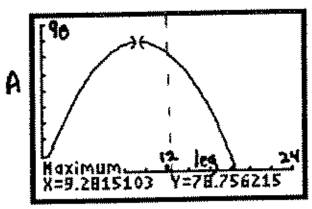


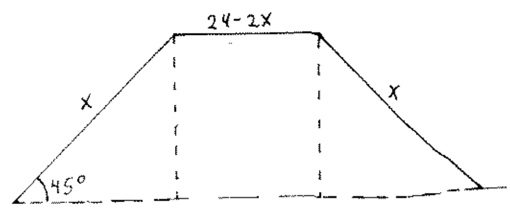
at x = 12 it becomes a triangle.



Isosceles trapezoid with 45° base angles.

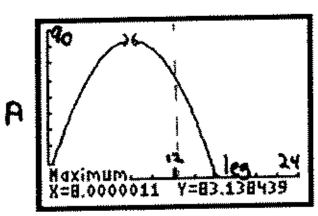
$$A(x) = (.5)(\frac{x\sqrt{2}}{2})(2(24-2x)+2(\frac{x\sqrt{2}}{2}))$$

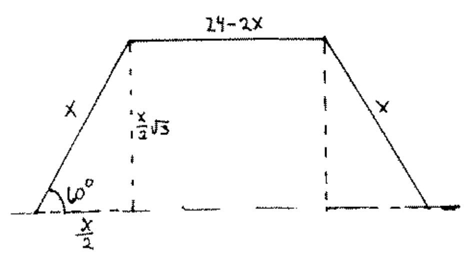




Isosceles trapezoid with 60° base angles.

$$A(x) = (.5)\ (\frac{x\sqrt{3}}{2})(2(24-2x)+2(\frac{x}{2}))$$





4. Examine a semi-circle: $A = (.5)\pi r^2$

$$r = \frac{24}{\pi}$$

$$A = \pi(\frac{24}{\pi})^2 = 91.67$$