Section 5.2:

#7. Perform Gram-Schmidt & find QR factorization of a matrix.

$\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}$

$$M = \begin{bmatrix} 2 & -2 & 18 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix} = QR = \begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix} \begin{bmatrix} 3 & 0 & 12 \\ 0 & 3 & -12 \\ 0 & 0 & 6 \end{bmatrix}$$

$\|\vec{v}_1\| = \sqrt{4+4+1} = \sqrt{9} = 3$

$$\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\vec{v}_2^{\perp} = \vec{v}_2 - (\vec{v}_2 \cdot \vec{u}_1)\cdot \vec{u}_1 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} - \Big(\underbrace{\tfrac{1}{3}\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}\cdot\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}_{=0}\Big)\cdot\frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$\|\vec{v}_2^{\perp}\| = \sqrt{4+1+4} = \sqrt{9} = 3$

$$\vec{u}_2 = \frac{\vec{v}_2^{\perp}}{\|\vec{v}_2^{\perp}\|} = \frac{1}{3}\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{v}_3^{\perp} = \vec{v}_3 - (\vec{v}_3 \cdot \vec{u}_1)\cdot\vec{u}_1 - (\vec{v}_3 \cdot \vec{u}_2)\cdot\vec{u}_2 = \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix} - \Big(\underbrace{\tfrac{1}{3}\begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}_{=12}\Big)\cdot\frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} - \Big(\underbrace{\tfrac{1}{3}\begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}\cdot\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}}_{=-12}\Big)\cdot\frac{1}{3}\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \\ 4 \end{bmatrix} + \begin{bmatrix} -8 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 4 \end{bmatrix}$$

$$\vec{u}_3 = \frac{\vec{v}_3^{\perp}}{\|\vec{v}_3^{\perp}\|} = \frac{1}{6}\begin{bmatrix} 2 \\ -4 \\ 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$\|\vec{v}_3^{\perp}\| = \sqrt{4+16+16} = \sqrt{36} = 6$

#18: Find QR factorization of $\begin{bmatrix} 4 & 25 & 0 \\ 0 & 0 & -2 \\ 3 & -25 & 0 \end{bmatrix} = M$

$\|\vec{v}_1\| = \left\|\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}\right\| = \sqrt{16+9} = \sqrt{25} = 5$

$$M = QR = \begin{bmatrix} 4/5 & 3/5 & 0 \\ 0 & 0 & -1 \\ 3/5 & -4/5 & 0 \end{bmatrix}\begin{bmatrix} 5 & 5 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\vec{v}_2^{\perp} = \vec{v}_2 - (\vec{v}_2 \cdot \vec{u}_1)\vec{u}_1 = \begin{bmatrix} 25 \\ 0 \\ -25 \end{bmatrix} - \Big(\underbrace{\tfrac{1}{5}\begin{bmatrix} 25 \\ 0 \\ -25 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}}_{=5}\Big)\cdot\frac{1}{5}\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} =$$

$$= \begin{bmatrix} 25 \\ 0 \\ -25 \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 21 \\ 0 \\ -28 \end{bmatrix}$$

& $\|\vec{v}_2^{\perp}\| = \sqrt{21^2 + 28^2} = \sqrt{441 + 784} = 35$

$\vec{v}_3$ is perpendicular to both $\vec{u}_1$ & $\vec{u}_2$, so we just need to normalize it

$\|\vec{v}_3\| = \sqrt{4} = 2$

#32 Find an orthonormal basis of the plane $x_1 + x_2 + x_3 = 0$.

Solution

First find any basis, then do Gram-Schmidt

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ belongs to the plane $\Leftrightarrow$ $x_1 + x_2 + x_3 = 0 \Leftrightarrow x_1 = -x_2 - x_3$

$$\Leftrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \text{Basis is } \Big\{ \underbrace{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{v}_1}, \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}_{\vec{v}_2} \Big\}$$

$$\|\vec{v}_1\| = \sqrt{2} \qquad \vec{u}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{v}_2^{\perp} : \vec{v}_2 - (\vec{v}_2 \cdot \vec{u}_1)\cdot \vec{u}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \Big(\underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}_{\frac{1}{\sqrt{2}}} \Big) \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix}$$

$$\|\vec{v}_2^{\perp}\| = \sqrt{\frac{1}{4} + \frac{1}{4} + 1} = \sqrt{\frac{3}{2}} \qquad \vec{u}_2 = \sqrt{\frac{2}{3}} \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}.$$

#39. Find an orthonormal basis $\vec{u}_1, \vec{u}_2, \vec{u}_3$ of $\mathbb{R}^3$ st $\text{Span}(\vec{u}_1) = \text{Span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$
$\text{span}(\vec{u}_1, \vec{u}_2) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\right)$

Solution

$\text{span}(\vec{u}_1) = \text{Span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$ & $\vec{u}_1$ is unit $\Rightarrow$ $\vec{u}_1 = \dfrac{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}{\left\|\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right\|} = \dfrac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\text{span}(\vec{u}_1, \vec{u}_2) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\right)$ & $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ are orthogonal $\Rightarrow$

$$\vec{u}_2 = \frac{\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}}{\left\|\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\right\|} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{u}_3 = \vec{u}_1 \times \vec{u}_2 = \frac{1}{\sqrt{42}} \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix}$$

Section 5.3:

If A & B are arbitrary $n\times n$ matrices which of the following must be symmetric?

#25: $(A^T B^T B A)^T = A^T B^T B^{TT} A^{TT} = A^T B^T B A$ Yes

#26: $(B(A+A^T)B^T)^T = B^{TT}(A+A^T)^T B^T = B(A^T+A)B^T$ Yes

#28: Consider an orthogonal transformation $L:\mathbb{R}^n \to \mathbb{R}^n$ Show that L preserves the dot product:

$$\vec{v}\cdot\vec{w} = L(\vec{v})\cdot L(\vec{w})$$

Solution.

$L(\vec{x}) = A\vec{x}$ where A is an orthogonal matrix.

$$L(\vec{v})\cdot L(\vec{w}) = (A\vec{v})\cdot(A\vec{w}) = (A\vec{v})^T(A\vec{w}) = \vec{v}^T \underbrace{A^T A}_{=I_n} \vec{w} = \vec{v}^T\vec{w} = \vec{v}\cdot\vec{w}$$

#30. $L:\mathbb{R}^m \to \mathbb{R}^n$ preserves length. What can you say about kernel of L? What is dimension of the image? What can you say about relationship between n & m? If A is a matrix of L what can you say about the columns of A? What is A^TA? What is AA^T? Illustrate your answers with an example where $m=2$ & $n=3$?

Solution:

- $\vec{x}\in \ker L \Leftrightarrow L(\vec{x}) = 0$. L preserves length $0 = \|L(\vec{x})\| = \|\vec{x}\| \Rightarrow$ $\Rightarrow \vec{x} = \vec{0} \Rightarrow \ker L = \{\vec{0}\}$
- By rank-nullity theorem $m = \underbrace{\dim\ker L}_{=0} + \dim \operatorname{Im} L = \dim \operatorname{Im} L$
- Since $\operatorname{Im} L \subseteq \mathbb{R}^n$ & $\dim \operatorname{Im} L = m$, we see that $\underline{m \le n}$.
- $A = \begin{bmatrix} L(\vec{e}_1) & L(\vec{e}_2) & \cdots & L(\vec{e}_m)\end{bmatrix}$ & L preserves length $\Rightarrow$ columns of A are unit vectors. In fact, L will map orthogonal vectors to orthogonal vectors, so the columns of A form an orthonormal basis of Im L.
- $A^TA = I_m$ (this follows from this observation).

  AA^T = matrix of the orthogonal projection onto Im A.
- $L:\mathbb{R}^2 \to \mathbb{R}^3$

$$L\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_1\\x_2\\0\end{bmatrix}$$

#36 Find an orthogonal matrix of the form:

$$\begin{bmatrix} 2/3 & 1/\sqrt{2} & a \\ 2/3 & -1/\sqrt{2} & b \\ 1/3 & 0 & c \end{bmatrix}$$

Solution Both columns & rows are unit vectors orthogonal to each other (if A is orthogonal, so is A^T).

$$1 = \left\| \begin{bmatrix} 1/3 \\ 0 \\ c \end{bmatrix} \right\| = \sqrt{\frac{1}{9} + c^2} \Rightarrow c^2 = \frac{8}{9} \Rightarrow c = \pm \frac{2\sqrt{2}}{3}$$

$$0 = \begin{bmatrix} 1/3 \\ 0 \\ c \end{bmatrix} \cdot \begin{bmatrix} 2/3 \\ -1/\sqrt{2} \\ b \end{bmatrix} = \frac{2}{9} + bc \Rightarrow b = -\frac{2}{9c} = -\frac{\cancel{2}}{3\,\cancel{9}\cdot\left(\pm\frac{\cancel{2}\sqrt{2}}{\cancel{3}}\right)} = \mp \frac{1}{3\sqrt{2}}$$

$$0 = \begin{bmatrix} 1/3 \\ 0 \\ c \end{bmatrix} \cdot \begin{bmatrix} 2/3 \\ 1/\sqrt{2} \\ a \end{bmatrix} = \frac{2}{9} + ac \Rightarrow a = -\frac{2}{9c} = b = \mp \frac{1}{3\sqrt{2}}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1/3\sqrt{2} \\ -1/3\sqrt{2} \\ 2\sqrt{2}/3 \end{bmatrix} \text{ or } \begin{bmatrix} 1/3\sqrt{2} \\ 1/3\sqrt{2} \\ -2\sqrt{2}/3 \end{bmatrix}$$

Section 5.4.

#9: Consider the linear system $A\vec{x} = \vec{b}$ where $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$.

a) Draw a sketch showing the following subsets of $\mathbb{R}^2$:

- $\ker A$ & $(\ker A)^\perp$
- $\operatorname{Im}(A^T)$
- the solution set S of the system $A\vec{x} = \vec{b}$

b) What relationship do you observe between $\ker A$ & $\operatorname{Im} A^T$?

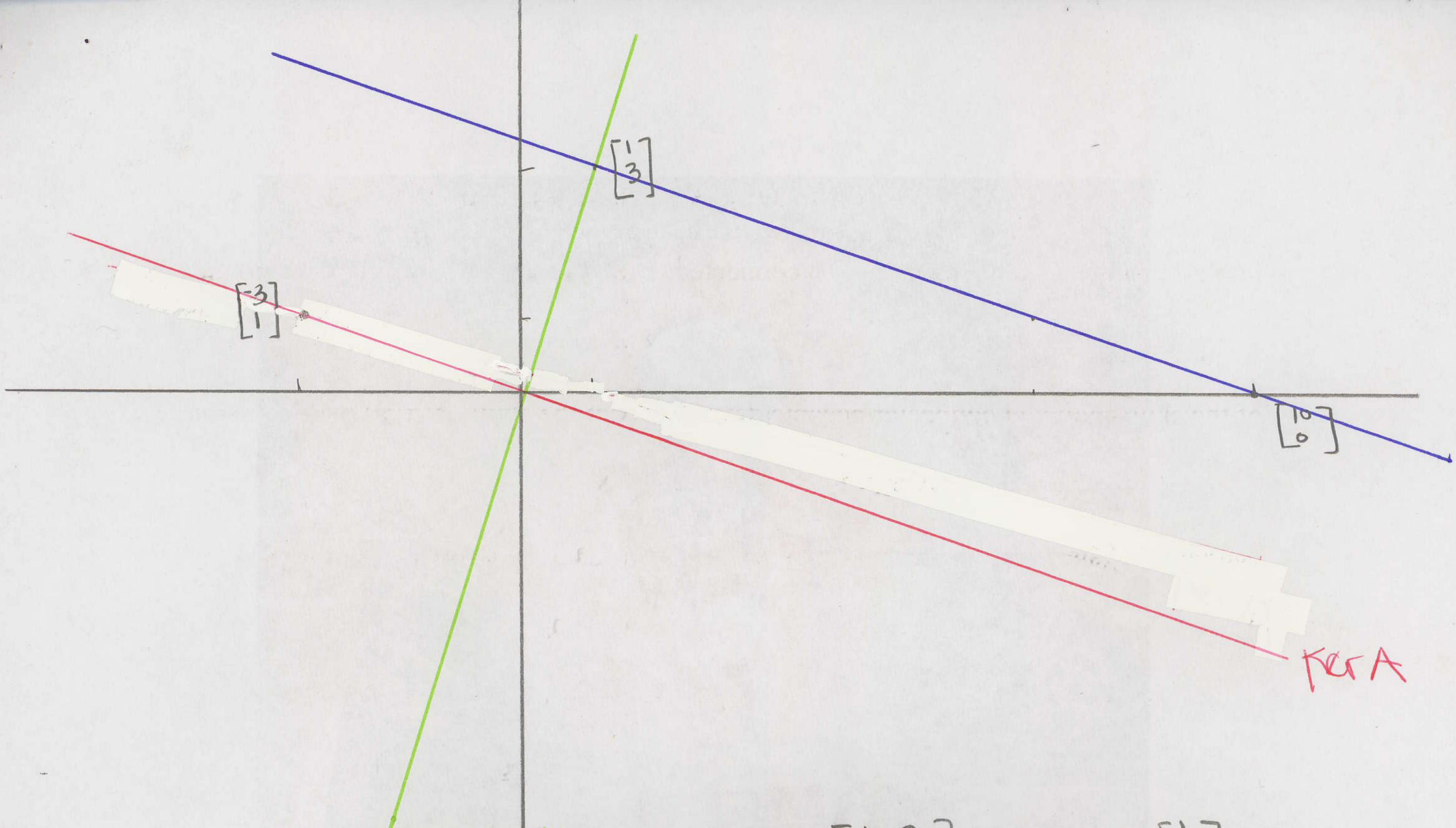
c) What relationship do you observe between $\ker A$ & S?

d) Find the unique vector $\vec{x}_0$ in the intersection of S and $(\ker A)^\perp$. Show $\vec{x}_0$ on your sketch.

e) What can you say about length of $\vec{x}_0$ compared with the length of all other vectors in S?

a) $A\vec{x} = \vec{0} \rightsquigarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$ $\quad x_1 = -3x_2$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \ker A \iff \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \quad x_2 \in \mathbb{R}$$



$(\ker A)^{\perp} = \operatorname{Im} A^{T} = \operatorname{Im}\left(\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\right) = \operatorname{Span}\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$

$$A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \rightsquigarrow \left[\begin{array}{cc|c} 1 & 3 & 10 \\ 2 & 6 & 20 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 3 & 10 \\ 0 & 0 & 0 \end{array}\right] \rightsquigarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 - 3x_2 \\ x_2 \end{bmatrix} = x_2\begin{bmatrix} -3 \\ 1 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$S = \left\{ \begin{bmatrix} 10 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix} : x_2 \in \mathbb{R} \right\}$$

b) $\ker A = (\operatorname{Im} A^{T})^{\perp}$

c) $\ker A$ & S are two parallel lines in $\mathbb{R}^2$. In fact $S = \ker A + \begin{bmatrix} 10 \\ 0 \end{bmatrix}$.

d) From the sketch we see that $\vec{x}_0 \in S \cap (\ker A)^{\perp}$. Algebraically:

$\vec{x}_0 = \begin{bmatrix} a \\ b \end{bmatrix} \in S \cap (\ker A)^{\perp} \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 - 3x_2 \\ x_2 \end{bmatrix}$, for some $x_2 \in \mathbb{R}$, since $\vec{x}_0 \in S$

& $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y \\ 3y \end{bmatrix}$, for some $y \in \mathbb{R}$, since $\vec{x}_0 \in (\ker A)^{\perp}$

$\Rightarrow \left.\begin{array}{l} 10 - 3x_2 = y \\ x_2 = 3y \quad /\cdot 3 \end{array}\right\} \quad \left.\begin{array}{l} 10 - 3x_2 = y \\ 3x_2 = 9y \end{array}\right\} + \quad 10 = 10y \Rightarrow y = 1 \Rightarrow$

$\vec{x}_0 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y \\ 3y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

e) Any other vector in S has length greater than that of $\vec{x}_0$.

#22. Find the least squares solution $\vec{x}^*$ of the system $A\vec{x} = \vec{b}$, where

$A = \begin{bmatrix} 3 & 2 \\ 5 & 3 \\ 4 & 5 \end{bmatrix}$ & $\vec{b} = \begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix}$. Determine the error $\|\vec{b} - A\vec{x}^*\|$.

We are looking for $\vec{x}^*$ st $A^TA\vec{x}^* = A^T\vec{b}$

$$A^TA = \begin{bmatrix} 3 & 5 & 4 \\ 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 50 & 41 \\ 41 & 38 \end{bmatrix}$$

$$A^T\vec{b} = \begin{bmatrix} 3 & 5 & 4 \\ 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix} = \begin{bmatrix} 68 \\ 47 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 50 & 41 & 68 \\ 41 & 38 & 47 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 41/50 & 68/50 \\ 0 & 219/50 & -438/50 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 41/50 & 68/50 \\ 0 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right] \Rightarrow$$

$$\vec{x}^* = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\|\vec{b} - A\vec{x}^*\| = \left\| \begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 5 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix} \right\| = 0$$

The system $A\vec{x} = \vec{b}$ was in fact consistent, i.e. $A\vec{x}^* = \vec{b}$, so $\vec{x}^*$ was the solution to $A\vec{x} = \vec{b}$.

#37:

plane	Year t	displays d
douglas DC-3	'35	35
Lockheed Constellation	'46	46
Boeing 707	'59	77
Concorde	'69	133

a) Fit a linear function of the form $\log(d) = c_0 + c_1 t$ to the data points $(t_i, \log(d_i))$ using least squares.

b) Use your answer in part a) to fit an exponential function $d = ka^t$ to the data points (t_i, d_i)

c) The airbus A320 was introduced in 1988. Based on your answer in b) how many displays do you expect in the cockpit of A320 (it had 93).

a) We need c_0 & c_1 st

$$c_0 + 35 \cdot c_1 = \log 35$$
$$c_0 + 46 \cdot c_1 = \log 46$$
$$c_0 + 59 \cdot c_1 = \log 77$$
$$c_0 + 69 c_1 = \log 133$$

$$\begin{bmatrix} 1 & 35 \\ 1 & 46 \\ 1 & 59 \\ 1 & 69 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \log 35 \\ \log 46 \\ \log 77 \\ \log 133 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 35 & 46 & 59 & 69 \end{bmatrix} \begin{bmatrix} 1 & 35 \\ 1 & 46 \\ 1 & 59 \\ 1 & 69 \end{bmatrix} = \begin{bmatrix} 4 & 209 \\ 209 & 11583 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 35 & 46 & 59 & 69 \end{bmatrix} \begin{bmatrix} 1.544 \\ 1.663 \\ 1.886 \\ 2.124 \end{bmatrix} = \begin{bmatrix} 7.217 \\ 388.36 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 4 & 209 & 7.217 \\ 209 & 11583 & 388.36 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 52.25 & 1.804 \\ 1 & 55.42 & 1.858 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 52.25 & 1.804 \\ 0 & 3.17 & 0.054 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 52.25 & 1.804 \\ 0 & 1 & 0.017 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 0.914 \\ 0 & 1 & 0.017 \end{array}\right] \Rightarrow \log d = 0.914 + 0.017\, t$$

b) $\log d = 0.914 + 0.017\, t$

$$d = 10^{0.914} \cdot 10^{0.017 t} = 8.2 \cdot 10^{0.017 t}$$

c) $$d_{\text{airbus A320}} = 8.2 \cdot 10^{0.017 \cdot 88} = 8.2 \cdot 10^{1.496} \approx 257$$