Homework 2
Due: Friday, 2/6/2004

1. If G acts on X and Y show that $g(x,y) = (gx, gy)$ defines an action of G on $X \times Y$. We call this diagonal action of G on $X \times Y$. What is the stabilizer of a point $(x,y) \in X \times Y$? If G acts transitively on both X and Y, is the diagonal action necessarily transitive?

2. Prove the following claims.
a) Product of two translations is a translation.
b) Product of two rotations is a rotation unless the sum of their angles is $0 \mod 2\pi$ in which case it is a translation.
c) Product of a translation and a nontrivial rotation is a rotation.
d) Product of a nontrivial translation and a reflection is a glide reflection, unless the axis of translation is perpendicular to that of a reflection in which case it is a reflection.
e) Product of a nontrivial rotation and a reflection is a glide reflection, except when the axis of reflection passes through the center of rotation in which case it is a reflection.

3) Prove that G is any subgroup of $\mathcal{E}$ then $|G : G^+|$ is 1 or 2. (G^+ denotes subgroup of G consisting of only orientation preserving isometries)

4) Show that for any subgroup G of $\mathcal{E}$, the requirement that $G \cap \mathcal{T} \cong \mathbb{Z}$ forces G to be discrete.