Homework 5
Due: Friday, 3/26/2004

1. Show that Möb($\mathbb{H}$) acts transitively on the set $\mathcal{H}$ of open half-planes in $\mathbb{H}$.

2. Given two pairs (z_1, z_2) and (w_1, w_2) of distinct points of $\mathbb{H}$, prove that there exists an element $m \in$ Möb($\mathbb{H}$) taking one to the other if and only if $d_{\mathbb{H}}(z_1, z_2) = d_{\mathbb{H}}(w_1, w_2)$.

3) Prove that every hyperbolic isometry takes hyperbolic lines to hyperbolic lines.

4) Let $x = \mu i$ and $y = \lambda i$ be two distinct points on the positive imaginary axis I. Let z be any point on I. Show that z is uniquely determined by hyperbolic distances $d_{\mathbb{H}}(x, z)$ and $d_{\mathbb{H}}(y, z)$.

5) We say that a subset X of the hyperbolic plane is *convex* if for each pair of points $x, y \in X$, the closed hyperbolic line segment ℓ_{xy} is contained in X. Show that half-planes in hyperbolic space are convex sets.