

### Homework 4

Due: Friday, 3/12/2004

1. Determine the Euclidean center and Euclidean radius of the image of the Euclidean circle  $A$  given by the equation  $\alpha z\bar{z} + \beta z + \bar{\beta}\bar{z} + \gamma = 0$  under  $f(z) = az + b$ ,  $f(\infty) = \infty$ , where  $a, b \in \mathbb{C}$ ,  $a \neq 0$ .
2. Let  $A$  be Euclidean circle in  $\mathbb{C}$  given by equation  $|z - z_0| = r^2$ . Determine conditions on  $z_0$  and  $r$  st  $J(A)$  is a Euclidean line in  $\mathbb{C}$ .
- 3) Consider the unoredered triple  $T = \{0, 1, \infty\}$  of points in  $\mathbb{C}$ . Determine all Möbius transformations  $m$  satisfying  $m(T) = T$ .
- 4) Give an explicit Möbius transformation taking  $D = \{z \in \mathbb{C} \mid |z| < 1\}$  to  $\mathbb{H}$ .
- 5) Normalize each of the following Möbius transformations and determine their types (elliptic, parabolic, loxodromic):

$$m(z) = \frac{-z - 3}{z + 1}$$

$$m(z) = \frac{iz + 1}{z + 3i}$$

$$m(z) = iz + 1$$