

Name. _____

Calculus III 2210-4
Sample Midterm Exam 1
Exam Date: Wed 5 October 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (Vector calculus) Complete two of the following.

(a) Let $\mathbf{r}(t) = \begin{pmatrix} t \\ 2t \end{pmatrix}$. Define $\mathbf{u}(t) = (\mathbf{r}(t) \cdot \mathbf{r}'(t))\mathbf{r}(3t + 1)$. Find $\mathbf{u}'(0)$.

(b) Let $\mathbf{r}(t) = \begin{pmatrix} \sin t \\ t \end{pmatrix}$. Find $\int_0^\pi \mathbf{r}(t) dt$.

(c) True or false: $\mathbf{u} \times \mathbf{v}$ is perpendicular to \mathbf{u} . Justify.

2. (Vector algebra) Complete two of the following.

(a) Report three different vectors orthogonal to $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(b) Find all vectors $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} - \mathbf{j}$.

(c) State the five dot product properties.

3. (Differential geometry) Complete two of the following.

(a) State a formula for the curvature of a space curve $\mathbf{r}(t)$. Use it to justify why $|\mathbf{r}'(t) \times \mathbf{r}''(t)| = 0$ characterizes zero curvature.

(b) Let $\mathbf{r}(t) = t\mathbf{i} + 2\mathbf{j} + (t + 1)\mathbf{k}$. Find the principal normal \mathbf{N} at $t = 0$.

(c) Let $\mathbf{r}(t) = t\mathbf{i} + (t+1)\mathbf{k}$. Find the tangential component of acceleration a_T at $t = 0$.

4. (Graphing)

Name and sketch the graph of $x^2 + y^2 - z^2 = 1$.

5. (Planes) Complete one of the following.

(a) Find the equation of the plane parallel to $x + 2y + 9z = 2$ passing through the point $(2, -2, 4)$.

(b) Find the acute angle between the planes $2x - 4y + z = 7$ and $3x + 2y - 5z = 9$.