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Calculus III 2210-4
Sample Midterm Exam 1
Exam Date: Wed 28 Sept 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. **(Vector calculus)**

   (a) Let $\mathbf{r}(t) = \begin{pmatrix} e^{2t} \\ e^{-t} \end{pmatrix}$. Define $\mathbf{u}(t) = r\mathbf{r}(t) + (\mathbf{r}(t) \cdot \mathbf{r}'(t))\mathbf{r}(2t+10)$. Find $\mathbf{u}'(0)$.

   (b) Let $\mathbf{r}(t) = \begin{pmatrix} e^{2t} \\ e^{-t} \\ e^{3t} \end{pmatrix}$. Find $\int_0^{\ln 2} \mathbf{r}(t)dt$.

   (c) State and prove a formula for the derivative of the cross product of two vector functions $\mathbf{u}(t)$ and $\mathbf{v}(t)$.

   (d) True or false: $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$. Justify.

   (e) True or false: $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{v})\mathbf{w} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$. Justify.

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2. (Vector algebra)

(a) Prove that orthogonal vectors $\mathbf{u}$, $\mathbf{v}$ satisfy $|\mathbf{u}|^2 + \mathbf{v}|^2 = |\mathbf{u} + \mathbf{v}|^2$.

(b) Report three different vectors orthogonal to $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(c) Let $\mathbf{u}$ and $\mathbf{v}$ have dot product zero. Prove that they are 90 degrees apart.

(d) For which vectors $\mathbf{u}$ and $\mathbf{v}$ is it true that $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$? Justify.

(e) State the five dot product properties.

(f) State the eight properties in the vector toolkit.

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3. (Differential geometry)

(a) Prove that a curve $y = f(x)$ with $f''(x) = 0$ has zero curvature.

(b) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the velocity and acceleration at $t = \ln 2$.

(c) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the unit tangent, principal normal, binormal and curvature at $t = \ln 2$.

(d) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the tangential and normal components of acceleration at $t = \ln 2$.

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4. (Graphing)
Name and sketch the graph of
(a) $3x^2 + 4y^2 + 9z^2 = 36$
(b) $x^2 + y^2 - z^2 = 1$
(c) $x + 2y + 5z = 3$
(d) $x^2 + z^2 = 4$

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5. (Planes)

(a) Find the equation of the plane parallel to $x + 2y + 9z = 2$ passing through the point $(2, -2, 4)$.

(b) Find the acute angle between the planes $2x - 4y + z = 7$ and $3x + 2y - 5z = 9$.

(c) Find the distance between the parallel planes $x + y + z = 5$ and $2x + 2y + 2z = 9$.

(d) Find the equation of the plane which passes through points $(1, 1, 1)$, $(1, 1, -2)$ and $(2, 1, 2)$.

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