Name. KEY

Calculus III 2210-4
Midterm Exam 1
Exam Date: Wed 5 October 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (Vector calculus) Complete two of the following.

(a) Let $\mathbf{r}(t) = \begin{pmatrix} t-1 \\ 4t \end{pmatrix}$. Define $\mathbf{u}(t) = (\mathbf{r}(t)\cdot\mathbf{r}'(t))\mathbf{r}(t^2+2t+1)$. Find $\mathbf{u}'(0)$.

(b) Let $\mathbf{r}(t) = \begin{pmatrix} \tan t \\ t-\pi \end{pmatrix}$. Find $\int_0^{\pi/4}\mathbf{r}(t)dt$.

(c) True or false: $\mathbf{u}\times\mathbf{v}$ can be the zero vector. Justify.

(a) $\vec{u}' = (\vec{r}'\cdot\vec{r}' + \vec{r}\cdot\vec{r}'')\vec{r}(t^2+2t+1) + (r\cdot r')\,r'(t^2+2t+1)(2t+2)$

$r' = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \quad r'' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\vec{u}'(0) = (1+16)\vec{r}(1) + (r\cdot r')r'(1)(2)$

$= 17\begin{pmatrix} 0 \\ 4 \end{pmatrix} + (-1)\begin{pmatrix} 1 \\ 4 \end{pmatrix}(2)$

$= \begin{pmatrix} -2 \\ 60 \end{pmatrix}$

(b) $\int_0^{\pi/4}\vec{r}\,dt = \begin{pmatrix} -\ln(\cos\frac{\pi}{4}) + \ln(\cos 0) \\ \frac{1}{2}(\frac{\pi}{4}-\pi)^2 - \frac{1}{2}(0-\pi)^2 \end{pmatrix}$

$= \begin{pmatrix} \ln\sqrt{2} \\ (\frac{9}{32}-\frac{1}{2})\pi^2 \end{pmatrix}$ also $\begin{pmatrix} \frac{1}{2}\ln 2 \\ \frac{-7}{32}\pi^2 \end{pmatrix}$

(c) True, let $\vec{u}=\vec{0}$ or $\vec{v}=\vec{0}$.

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2. (Vector algebra) Complete two of the following.

(a) Report two different vectors orthogonal to the plane $x+y+2z=0$ and the vector $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(b) Find all vectors $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ orthogonal to both $\mathbf{i}+\mathbf{k}$ and $\mathbf{j}+\mathbf{k}$.

(c) State five of the eight vector toolkit properties.

(a) $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \perp$ plane; need $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

or $a+b+2c = 0 = a-b+2c$. Choose $\boxed{\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}}$.

(b) Need $v \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 0 = v \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ or $a+c=0=b+c$

Then $a=b=-c$. ans: $\boxed{c \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}$

(c)

$u+v=v+u$	comm
$u+(v+w)=(u+v)+w$	assoc
$u+\vec{0} = \vec{u}$	zero vector $\vec{0}$
$u+(-\vec{u}) = \vec{0}$	additive inverse $-\vec{u}$
$1 \cdot \vec{u} = \vec{u}$	

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3. (Differential geometry) Complete two of the following.

(a) State a formula for the curvature of a planar curve $y = f(x)$. Use it to justify why $f''(t) = 0$ characterizes zero curvature.

(b) Let $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + (1-2t)\mathbf{k}$. Find the principal normal $\mathbf{N}$ at $t = 0$.

(c) Let $\mathbf{r}(t) = t^2\mathbf{i} + (t+1)\mathbf{j} + t\mathbf{k}$. Find the normal component of acceleration a_N at $t = 0$.

(a) $\kappa = \dfrac{|y''|}{(1+y'^2)^{3/2}}$. Then $\kappa = 0 \Leftrightarrow y'' = 0$

(b) $r'' = a_T \vec{T} + a_N \vec{N}$

$\vec{r}(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\vec{r}' = \begin{pmatrix} 2t \\ 1 \\ -2 \end{pmatrix}$, $\vec{r}'(0) = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$, $\vec{r}'' = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$a_T = r'' \cdot \dfrac{r'}{|r'|} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \dfrac{1}{\sqrt{5}} = 0$

$\vec{N} = \dfrac{r''}{a_N}$. Because $|\vec{N}| = 1$, Then $\boxed{\vec{N} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$

(c) $a_N = \dfrac{|r' \times r''|}{|r'|}$ $\quad r' = \begin{pmatrix} 2t \\ 1 \\ 1 \end{pmatrix}$ $\quad r'' = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$r' \times r'' = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{vmatrix}$

$= \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$

$|r' \times r''| = \sqrt{8}$

$|r'| = \sqrt{2}$

$\boxed{a_N = 2}$

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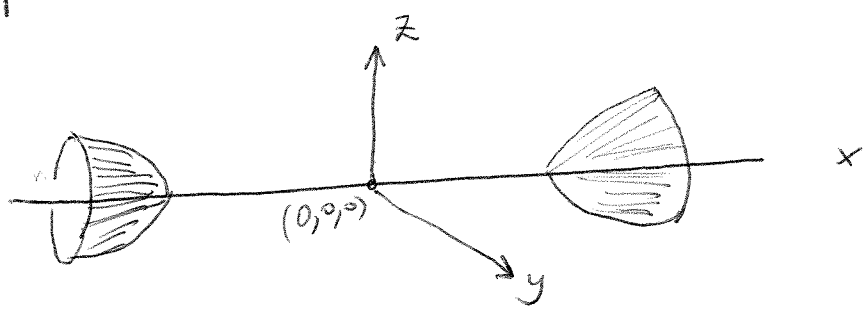
4. (Graphing)
Name and sketch an approximate graph for $x^2 - 3y^2 - z^2 = 1$.

Hyperboloid of two Sheets



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5. (Planes) Complete one of the following.
(a) Find the equation of the plane passing through the point $(2, -2, 4)$ whose normal is parallel to the line of intersection of the two planes $x+2y+9z = 2$ and $x+y+3z = 1$.
(b) Find the cosine of the angle θ between the plane $2x + y + z = 4$ and the plane determined by the three points $(1, 1, 0)$, $(0, 1, 1)$ and $(-1, 0, 1)$.

(a) $\vec{n}_1 = \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix}$, $\vec{n}_2 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 9 \\ 1 & 1 & 3 \end{vmatrix} = \begin{pmatrix} -3 \\ 6 \\ -1 \end{pmatrix}$$

$$\boxed{(-3)(x-2) + (6)(y-(-2)) + (-1)(z-4) = 0} \quad \text{or} \quad \boxed{3x - 6y + z = 22}$$

(b) $$\cos(\theta) = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{6}\sqrt{3}} = \frac{\sqrt{2}}{3}$$

$\vec{n}_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$$\vec{n}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

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