Name. KEY

2210-4 Midterm 3

1. (Space Geometry) Complete the following.

(a) [50%] Find the directional derivative of $f(x,y) = x^2 - 3xy + 2y^2$ at $(-1,2)$ in the direction of $2\mathbf{i} - \mathbf{j}$.

(b) [50%] Find a point on the surface $z = 2x^2 + 3y^2$ where the tangent plane is parallel to the plane $8x - 3y - z = 0$.

ⓐ $$\text{grad}(f) = \begin{pmatrix} 2x - 3y \\ -3x + 4y \end{pmatrix}\Bigg|_{\substack{x=-1 \\ y=2}}$$

$$= \begin{pmatrix} -8 \\ 11 \end{pmatrix}$$

$$\text{D.D.} = \begin{pmatrix} -8 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} = \boxed{-27/\sqrt{5}}$$

ⓑ $$\text{grad } f = \begin{pmatrix} 4x \\ 6y \\ -1 \end{pmatrix} \quad \text{for } f = 2x^2 + 3y^2 - z$$

plane normal is $\vec{n} = \begin{pmatrix} 8 \\ -3 \\ -1 \end{pmatrix}$. Then $\text{grad } f = c\vec{n} \iff$

$c = 1$ and $4x = 8$, $6y = -3$.

ans: $x = 2$, $y = -1/2$, $z = 2x^2 + 3y^2 = 8 + 3/4$

$$\boxed{\left(2, -1/2, \underset{\substack{\uparrow \\ 35/4}}{8.75}\right)}$$

Use this page to start your solution. Attach extra pages as needed, then staple.

Name. KEY 2210-4 Midterm 3

2. (Maxima)

Find the global maximum of $f(x, y) = x^2 - y^2 + 1$ on the circular disk $x^2 + y^2 \le 1$.

The max occurs, because a continuous $f(x,y)$ on a closed and bounded xy-set D has a max on D.

The max occurs on the boundary $x^2+y^2=1$, in which case $f = x^2-(1-x^2)+1 = 2x^2$, or else the max occurs at (x,y) satisfying $x^2+y^2<1$ (interior) and $f_x=0$, $f_y=0$.

$$f_x = 2x$$
$$f_y = -2y$$

The value at $x=y=0$ is $f=1$.
On the boundary, $f=2x^2$ and max $f=2$ occurs at $x=\pm 1$.

$\boxed{\max f = 2 \text{ at } (1,0) \text{ and } (-1,0).}$

Use this page to start your solution. Attach extra pages as needed, then staple.

Name. KEY

2210-4 Midterm 3

3. (Double Integral Theory) Complete the following.

(a) [50%] Define the double integral of $f(x,y)$ over a rectangle Q.

(b) [50%] Let region R be the union of three disjoint disks, two of which have areas 4 and one has area 5. Find the value of $\int\int_R dA$.

(a) $\iint_Q f\,dA$ = limit of Riemann Sums as $n \to \infty$ and mesh $\to 0$

R. sum $= \sum_{i=1}^{n} f(P_i)\,\Delta S_i$

P_i = center of subrectangle S_i

$S_1, \ldots, S_n$ are rectangles that have union Q

mesh = max diameter of any S_i

(b) $\iint_R dA = \iint_{R_1} dA + \iint_{R_2} dA + \iint_{R_3} dA$ where R = union of R_1, R_2, R_3

$= \text{area}(R_1) + \text{area}(R_2) + \text{area}(R_3)$

$= 4 + 4 + 5$

$= \boxed{13}$

Use this page to start your solution. Attach extra pages as needed, then staple.

Name. KEY

2210-4 Midterm 3

4. (Double Integrals) Complete the following.

(a) [50%] Find the volume of the solid in the first octant bounded by the cylinders $x^2 + z^2 = 16$, $y^2 + z^2 = 16$ and the coordinate planes. See 16.3-30

(b) [50%] Let R be the circular disk bounded by $x^2 + y^2 = 4$. Evaluate $\int\int_R f(x,y)dA$ using polar coordinates, given $f(x,y) = e^{x^2+y^2}$. See 16.4-11

(a) By symmetry, $vol = 2\int_0^4 \int_0^x (16-x^2)^{1/2} dy\,dx$

$= 2\int_0^4 x(16-x^2)^{1/2} dx$

$= \boxed{\frac{128}{3}}$ by the power rule

(b) $\iint_R e^{x^2+y^2} dA = \iint_{R^*} e^{r^2} r\,dr\,d\theta$

$= 2\int_0^{\pi} \int_0^2 e^{r^2} r\,dr\,d\theta$

$= \int_0^{\pi} (e^4 - 1)\,d\theta$

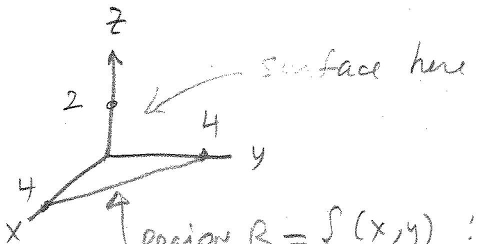
$= \boxed{\pi(e^4-1)}$

Use this page to start your solution. Attach extra pages as needed, then staple.

Name. KEY

2210-4 Midterm 3

5. (Surface Area)
Find the area of the part of the conical surface $x^2 + y^2 = z^2$ directly above the xy-plane triangle with vertices $(0, 0)$, $(4, 0)$ and $(0, 4)$. Include a figure [20%], inequalities for the region [20%] and integration details [60%]



Region $R = \{(x,y) : 0 \le x \le 4,\ 0 \le y \le 4-x\}$

$$A(S) = \int_0^4 \int_0^{4-x} \sqrt{1 + f_x^2 + f_y^2}\, dy\, dx$$

$$= \int_0^4 \int_0^{4-x} \sqrt{2}\, dy\, dx$$

$$= \boxed{8\sqrt{2}}$$

$$f = (x^2+y^2)^{1/2}$$

$$f_x = x(x^2+y^2)^{-1/2} = x/z$$

$$f_y = y(x^2+y^2)^{-1/2} = y/z$$

$$\sqrt{1+f_x^2+f_y^2} = \sqrt{1 + \frac{x^2+y^2}{z^2}}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

Use this page to start your solution. Attach extra pages as needed, then staple.