DYSFUNCTIONAL FAMILIES:

The Conic Sections

By
Marilyn Keir
Mathematics Department
University of Utah
keir@math.utah.edu

A CONICOPIA OF FUN!!

This can take as long as a month, or be shortened as time permits.

Prior to going into the unit, it is fun to hand out a list of the vocabulary words from the conics. There are so many interesting, new words. The students make daffynitions for these words.

CIRCLES:

DAY 1: The circle. Students in pairs draw a circle using nothing but their pencil and paper. Students must place axes on their paper, and write the equation and label the parts. Awards for the most perfect circle, largest radius, Pythagoras relationship are given.

DAY 2: The equation of a circle with which they are familiar is discussed in Standard form and General form and they are reminded of how to complete the square to get from general to standard.

ELLIPSES:

DAY 1: The ellipse is made with string using two paper fasteners as the foci. I allow them to put the center on the origin for ease of writing the equation. They make it on color paper and label it and doll it up for fun. Awards are given out.

DAY 2: The equation of an ellipse is discussed. Homework involves graphing and identifying parts of the equation in standard form.

DAY 3: Students must complete the square to get the equation from general to standard form, then graph and identify the parts.

DAY 4: Students paper fold the ellipse on graph paper. This time, they must put the center away from the origin as the instructions dictate. Label, etc.

DAY 5: Wrap up the ellipse stuff and go outside to do the human ellipse and the human circle using very long pieces of string.

HYPERBOLAS:

DAY 1: The hyperbola is made with string using two paper fasteners as the foci. I allow them to put the center on the origin for ease of writing the equation. They make it on color paper and label it and doll it up for fun. Awards are given out.

DAY 2: The equation of a hyperbola is discussed. Homework involves graphing and identifying parts of the equation in standard form.

DAY 3: Students must complete the square to get the equation from general to standard form, then graph and identify the parts.

DAY 4: Students paper fold the hyperbola on graph paper. This time, they must put the center away from the origin as the instructions dictate. Label, etc.

DAY 5: Wrap up the hyperbola stuff and go outside to do the human hyperbola using very long pieces of string.

PARABOLAS:

DAY 1: The parabola is made with string using two paper fasteners as the foci. I allow them to put the vertex on the origin for ease of writing the equation. They make it on color paper and label it and doll it up for fun. Awards are given out.

DAY 2: The equation of a parabola as a conic is discussed. Homework involves graphing and identifying parts of the equation in standard form.

DAY 3: Students must complete the square to get the equation from general to standard form, then graph and identify the parts.

DAY 4: Students paper fold the parabola on graph paper. This time, they must put the vertex away from the origin as the instructions dictate. Label, etc.

DAY 5: Wrap up the parabola stuff and go outside to do the human parabola using very long pieces of string.

ALGEBRUSH

Students will sketch a picture made up mostly of the basic conic shapes. They will write the equations for their picture. Students will spend two days in the computer lab using Algebrush to verify and enhance their pictures. Pictures will be printed and beautified by the students.

GROUP CONIC NAME____

CONIC COVER SHEET **Names:**__________________ __________________

Pd:____

Write your **score** for each item in this table:

CONIC	CIRCLE	ELLIPSE	HYPERBOLA	PARABOLA	Point totals
PASTE-UPS Two of each conic pasted on graph paper and equations written for each.	free-hand circle	string-drawn ellipse	string-drawn hyperbola	string-drawn parabola	______/50
		paper-fold ellipse	paper-fold hyperbola	paper-fold parabola	______/30
equation of conic graphed on calculator					______/20
MISC. PROBLEMS: Even by ________ Odd by ________					______/40
Misc other scores:	DAFFYNITIONS	Cover sheet			______/20
STICKERS FOR EXCELLENCE					Total points: ________/160 Total stickers: ________

- Packet should be in order, well-labeled and neatly done.
- Graph paper must be used on all assignments and on paper-fold items.
- The 4 pictures must be consistently labeled in this manner:

Conic name equations (all that apply)	a,b,c,e,p, r, area circumference (those which apply) found by measuring, not calculation
ERROR CHECK Pythagoras check or compare LR to 4P	Written definition & verification of definition using P_1, P_2, P_3

DIRECTIONS TO PAPERFOLD AN ELLIPSE

1. Write name in upper right hand corner of paper.

2. Label a point in the center of your paper F_1. With that point as a center, draw a large circle which nearly fills the page.

3. Place another point away from F_1, but neither horizontal or vertical to F_1. Label it O.

4. Place another point inside your circle **directly in a vertical or horizontal line with your F_1, an even number of squares away.** Call this point F_2.

5. Fold all points on the circle to F_2 until you have gone all the way around it.

6. Label the resulting conic and write the equation using O as the origin. (Draw in the axes.)

7. Label three points on your conic P_1 and P_2 and P_3. Measure the distance to each of the two foci. Use squares on the graph paper as units of measure. Add these distances. Does it fit the definition?

8. Write all of this information in the proper quadrant:

Quadrant II CONIC NAME Equation (standard) Equation (general)	Quadrant I a = b = c = (measured values) Area = e =
Quadrant III Pythagoras: Compare $a^2 = b^2 + c^2$	Quadrant IV Distances: $P_1F_1 + P_1F_2$ $P_2F_1 + P_2F_2$ $P_3F_1 + P_3F_2$ Definition in words

DIRECTIONS TO PAPERFOLD A HYPERBOLA

1. Write name in upper right hand corner of paper.

2. Place a point in the center of your paper. Label it O.

3. Label a point several squares away from the center of your paper F_1. With that point as a center draw a small circle (radius = 8 - 20) .

4. Place another point outside your circle **directly in a vertical or horizontal line with your F_1 , an even number of squares away.** Call this point F_2.

5. Fold all points on the circle to F_2 until you have gone all the way around it.

6. Label the resulting conic and write the equation using O as the origin. (Draw in the x and y axis.)

7. Label three points on your conic P_1 and P_2 and P_3. Measure the distance to each of the two foci. (Use squares on the graph paper as units of measure.) Subtract these distances and take the absolute value. Does it fit the definition?

8. Write all of this information in the proper quadrant:

Quadrant II CONIC NAME Equation (standard) Equation (general) Equation- asymptotes	Quadrant I a = b = c = (measured values) e =
Quadrant III Pythagoras: Compare c and a + b	Quadrant IV Distances: $\lvert P_1F_1 - P_1F_2 \rvert$ $\lvert P_2F_1 - P_2F_2 \rvert$ $\lvert P_3F_1 - P_3F_2 \rvert$ Definition in words

INSTRUCTIONS TO PAPERFOLD A PARABOLA

1. Write name in upper right hand corner of paper.

2. Draw a line across your paper about 2 or 3 squares from any of the four sides of the paper. Label it d.

3. In line with the center of the above line and about 4-12 (an even number) squares away from it, place a point. Label it F.

4. Place another point not on the line and neither horizontal nor vertical to F. Label it O.

5. Now place one end of your line on top of point F and crease the paper.

6. Continue to place points of your line (about 1 square apart) on top of F, and crease it. This will result in a parabola.

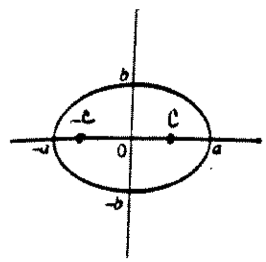
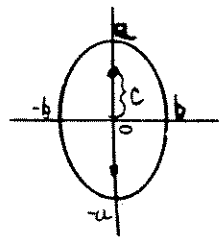
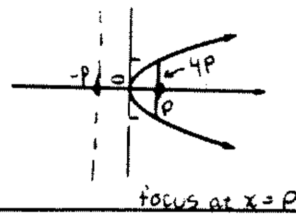
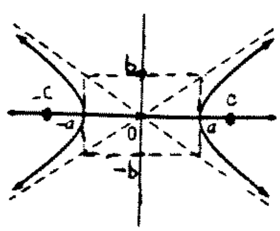
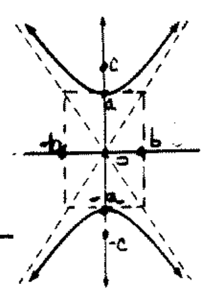
7. Using point 0 as the origin, write the equation of your parabola. Label the focus (F), the directrix, the axis of symmetry, and latus rectum. Check to see that the length of the l.r. is 4p.

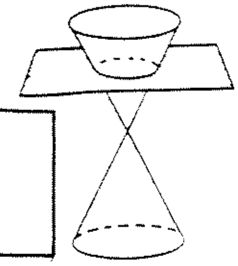
8. On the parabola, label three points, P_1, P_2, and P_3. Measure the distance of each of these from the directrix and the distance from the focus. (Use squares on the graph paper as units of measure.) What do you find?

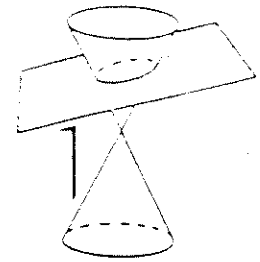
9. Write all of this information in the proper quadrant:

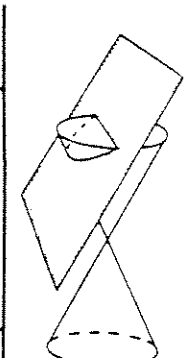
Quadrant II CONIC NAME Equation (standard) Equation (general) Equation of directrix Equation of axis of symmetry	Quadrant I p= LR = e = (measured values)
Quadrant III Compare 4p and LR	Quadrant IV distances: $P_1$to d = P_1F $P_2$to d = P_2F $P_3$to d = P_3F

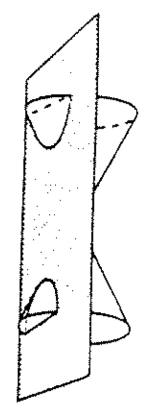
CONIC SUMMARY SHEET

circle	$a = b = r$ $e = 0$		$(x-h)^2 + (y-k)^2 = r^2$
horizontal ellipse $a \geq b$	$a^2 = b^2 + c^2$ $e = \dfrac{c}{a}$ $0 < e < 1$		$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
vertical ellipse $a \geq b$	$a^2 = b^2 + c^2$ $e = \dfrac{c}{a}$ $0 < e < 1$		$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$
parabola horizontal vertical	$e = 1$		$(y-k)^2 = 4p(x-h)$ $(x-h)^2 = 4p(y-k)$
horizontal hyperbola	$c^2 = a^2 + b^2$ $e = \dfrac{c}{a}$ $e > 1$		$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
vertical hyperbola	$c^2 = a^2 + b^2$ $e = \dfrac{c}{a}$ $e > 1$		$-\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$









CONIC PRACTICE

All work must be done neatly on graph paper with each problem clearly labeled. Do not change the scale. Show all work near the graph of the conic. For each conic you should identify those which apply: a, b, c, r, p, **e**. If there are any asymptotes, an axis of symmetry or a directrix, write the equation(s).

CIRCLE: Put in standard form and graph showing the location of the center.

1. $(x-3)^2 + (y+2)^2 = 36$
2. $(x+1)^2 + (y+2)^2 = 1$
3. $x^2 - 4x + y^2 = 21$
4. $x^2 + y^2 + 8y = -12$
5. $x^2 + y^2 - 20x + 20y = -100$
6. $x^2 + y^2 - 10x - 14y = -25$
7. $16x^2+16y^2+64x-32y = 176$
8. $36x^2+36y^2-72x+144y = 144$
9. Write the equation of a circle with center (-2, 4) which passes through the point (4, 4).
10. Write the equation of a circle with center (3, -2) which passes through the point (-5, 5).

ELLIPSE: Put each equation in standard form and sketch an accurate drawing, including the position of the foci.

11. $\frac{x^2}{25} + \frac{y^2}{4} = 1$
12. $\frac{x^2}{9} + \frac{y^2}{49} = 1$
13. $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{16} = 1$
14. $\frac{(x+1)^2}{25} + \frac{(y-2)^2}{4} = 1$
15. $4x^2 + y^2 - 32x + 16y + 124 = 0$
16. $2x^2 + 3y^2 + 12x - 24y + 60 = 0$
17. $9x^2+16y^2+54x-32y-47=0$
18. $9x^2+4y^2-18x+16y - 11 = 0$
19. Write the equation of an ellipse with center at (3, -5), a major axis length 10 and c = 3 with a horizontal direction to it.
20. Write the equation of an ellipse with center at (-2, -3), vertices at (-2, 2) and (-2, -7) and c = 4.

HYPERBOLA: Put each equation in standard form and sketch an accurate drawing, including the position of the foci.

21. $-\frac{x^2}{16} + \frac{y^2}{4} = 1$

22. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

23. $\frac{(x+1)^2}{9} - \frac{(y+2)^2}{4} = 1$

24. $-\frac{(x-2)^2}{1} + \frac{(y-2)^2}{4} = 1$

25. $9x^2 - 4y^2 - 36x + 8y - 4 = 0$

26. $25y^2 - 9x^2 - 54x - 50y - 281 = 0$

27. $4y^2 - 9x^2 - 18x - 8y - 41 = 0$

28. $16x^2 - y^2 - 32x - 6y - 57 = 0$

29. Write an equation of a hyperbola with center at (-2, 3) and a transverse axis of length 8 and conjugate axis length 6 which opens up and down.

30. Write an equation of a hyperbola with center (-4, -3) which opens left and right and has transverse axis length 4 and conjugate axis length 10.

PARABOLA: Put each equation in standard form and graph it, showing the focus, directrix , axis of symmetry and latus rectum.

31. $(y+4)^2 = -12(x+3)$

32. $(x+4)^2 = -8(y-2)$

33. $x^2 + y + 3 = 0$

34. $3x^2 - 6y + 12 = 0$

35. $y^2 - 4y - 8x + 20 = 0$

36. $y^2 - 2y + 4x - 11 = 0$

37. $x^2 - 6x + 8y - 7 = 0$

38. $y^2 - 4x - 6y + 5 = 0$

39. Write an equation of a parabola with the vertex at (-2, 3) and the axis of symmetry y=3 and p = -2.

40. Write the equation of a parabola with the vertex at (4, -1) and directrix y=3.

DAFFYNITIONS NAMES________________ ________________

You and your partner may brainstorm with another partnership, but each partnership should hand in a set of daffynitions.
The **CONICS** are filled with some strange new vocabulary words. Write a DAFFYNITION for each of these words. A daffynition uses the sound of the word in a humorous way. Be creative, but proper.

Example: HYPERBOLA -- A snake (boa) who had too many Cokes(thus, hyper.)

CONIC

VERTEX

ELLIPSE

MATH

FOCUS

CIRCLE

TRANSVERSE AXIS

CONE

RADIUS

ASYMPTOTES

CONJUGATE AXIS

FOCI

CENTER

QUADRATIC

DIRECTRIX

STANDARD EQUATIONS

AXIS OF SYMMETRY

BRANCHES

PARABOLA

MAJOR AXIS

MINOR AXIS

ECCENTRICITY

VERTICES

HYPERBOLA

ALGEBRUSH by MAREWARE

Algebrush is an MS-DOS (R) program which provides help to advanced algebra students (grades 9-12) as their understanding of functions and other relations deepens.

In typical classroom use, the student creates a picture by graphing selections from the 22 built-in functions and relations, controlling size and placement by assigning values to the domain, scaling and offset of each selection.

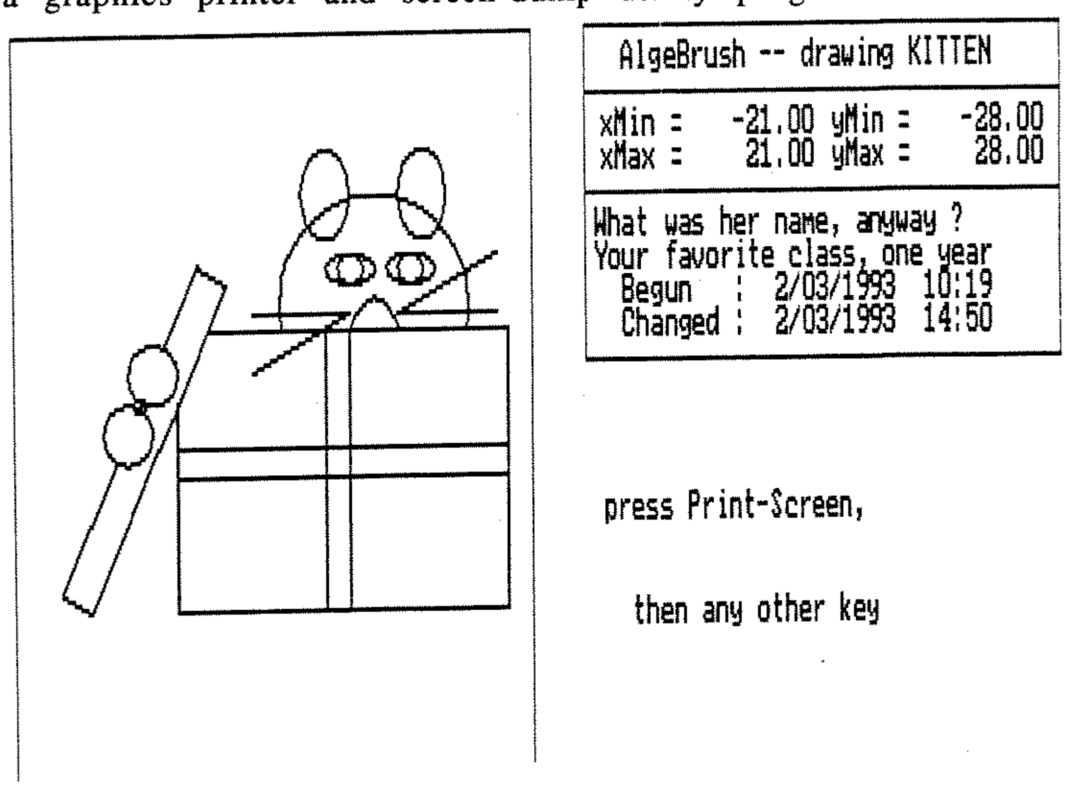
During construction, the picture can be viewed on-screen. The completed picture can be printed on a standard graphics printer. A list of the functions and values used by the student to create the picture can also be printed.

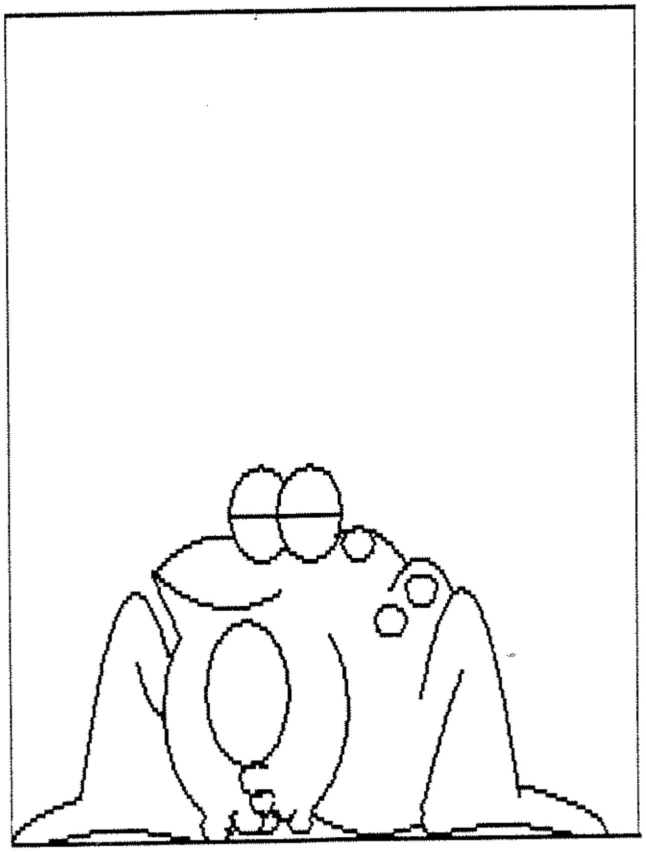
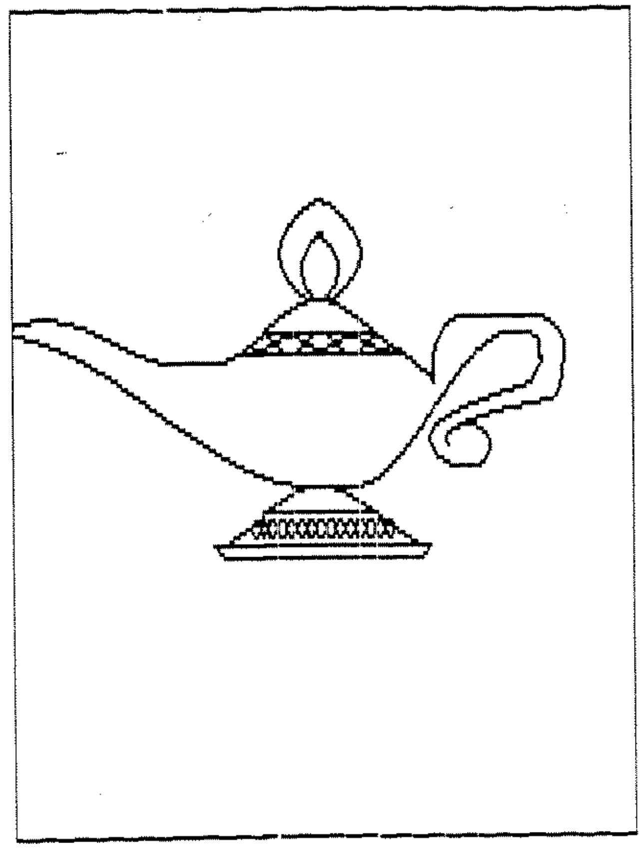
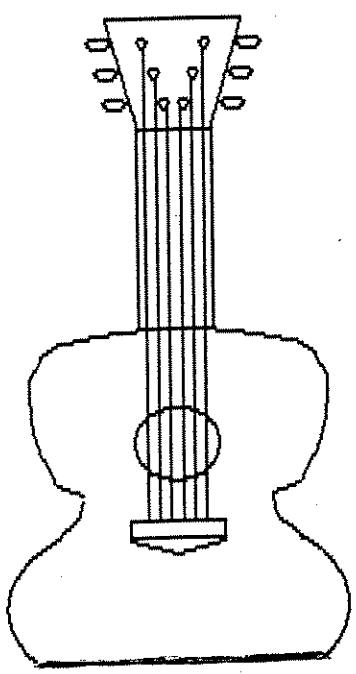
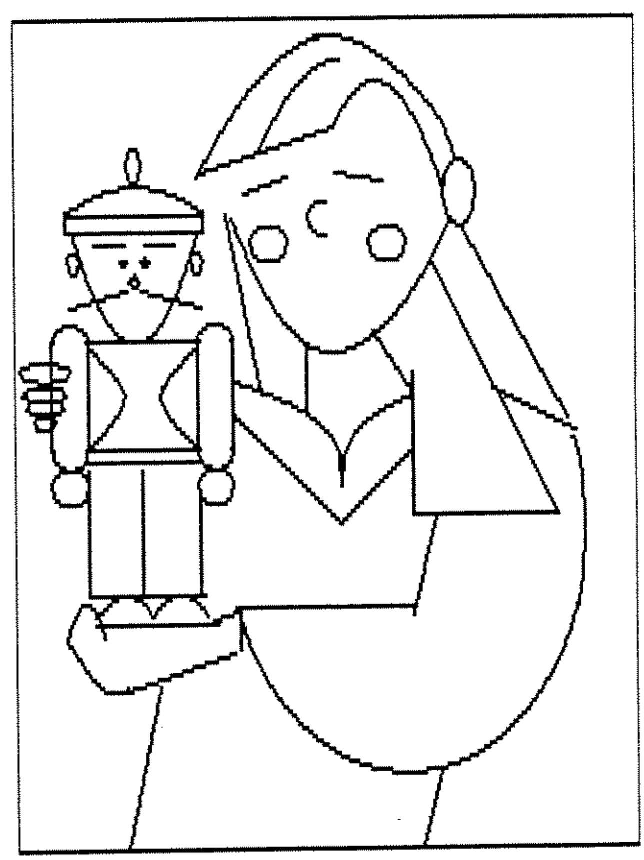
For more information, write to:

Mareware
530 Northmont Way
Salt Lake City, UT 84103

Program requirements :

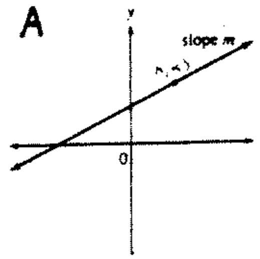
- MS-DOS, v3.2 or later.
- PC-compatible (XT or AT) with at least 512 KB of RAM.
- Graphics card and monitor (CGA, MCGA and VGA currently supported).
- Optional printer: The list output form can use any printer that responds to the MS-DOS PRINT command. The picture output form requires a graphics printer and screen-dump utility program.

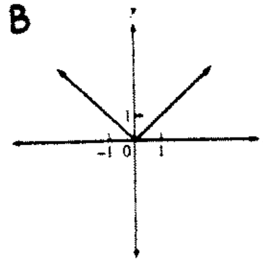


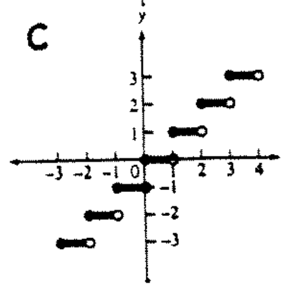





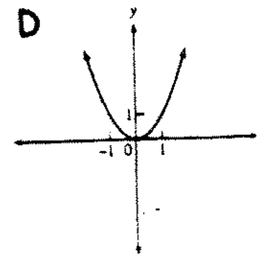
AlgeBrush Functions

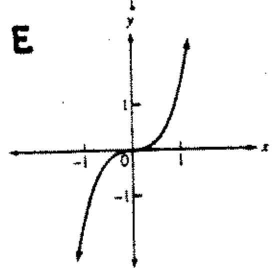
	FUNCTION or RELATION	EQUATION FORM
A	linear	$y = m(x-h) + k$
B	absolute value	$y = m\|x-h\| + k$
C	integer	$y = a \text{ INT } [s(x-h)] + k$
D	quadratic	$y = a(x-h)^2 + k$
E	cubic	$y = a(x-h)^3 + k$
F	reciprocal ($\frac{1}{x}$)	$y = \frac{a}{x-h} + k$
G	recip square ($\frac{1}{x^2}$)	$y = \frac{a}{(x-h)^2} + k$
H	square root	$y = a\sqrt{s(x-h)} + k$
I	cube root	$y = a\sqrt[3]{s(x-h)} + k$
J	exponential	$y = a e^{s(x-h)} + k$
K	logarithmic	$y = a \ln(s(x-h)) + k$
L	sine	$y = a \sin(s(x-h)) + k$

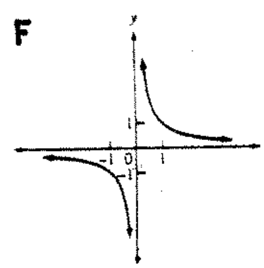
A


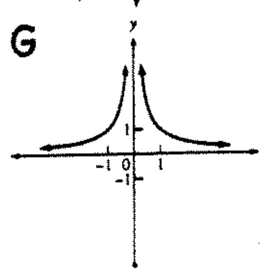
B


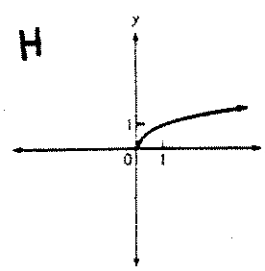
C


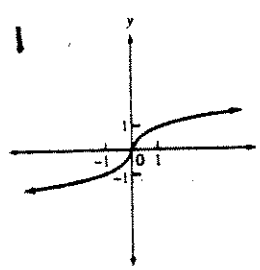
D


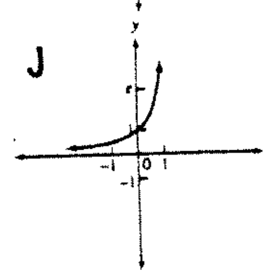
E


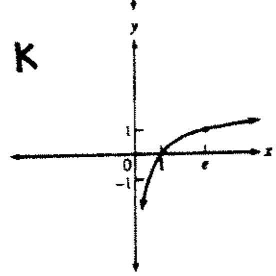
F


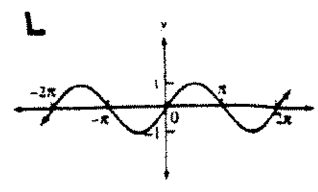
G


H


I


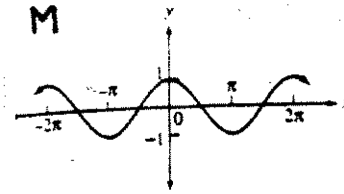
J


K


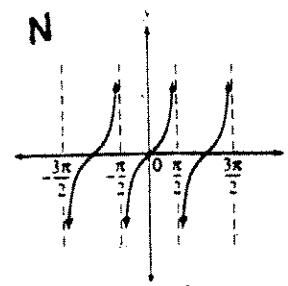
L


M	cosine	$y = a\cos(s(x-h)) + k$
N	tangent	$y = a\tan(s(x-h)) + k$
O	arcsine	$y = a\sin^{-1}(s(x-h)) + k$
P	arccosine	$y = a\cos^{-1}(s(x-h)) + k$
Q	arctangent	$y = a\tan^{-1} b(x-h) + k$
R	vertical line	$x = h$
S	circle	$(x-h)^2 + (y-k)^2 = r^2$
T	ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
U	sideways parabola	$x = b(y-k)^2 + h$
V	hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
W	updown hyperbola	$-\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
X	$x = \sin(y)$	$x = b\sin(s(y-k)) + h$
Y	$x = \text{int}(y)$	$x = b\,\text{INT}\,[s(y-k)] + h$

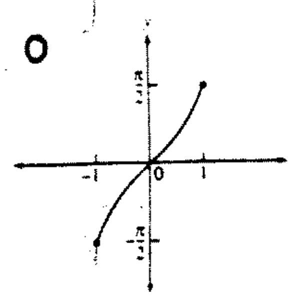
M



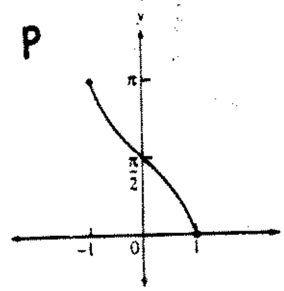
N



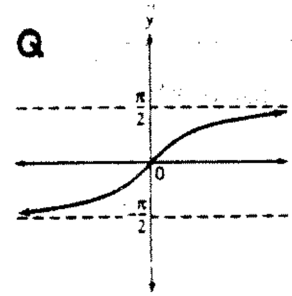
O



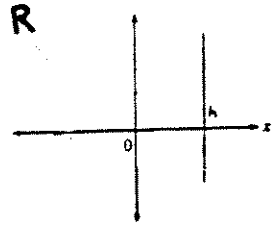
P



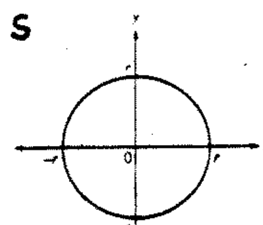
Q



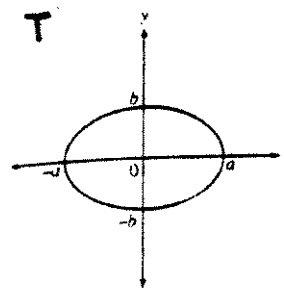
R



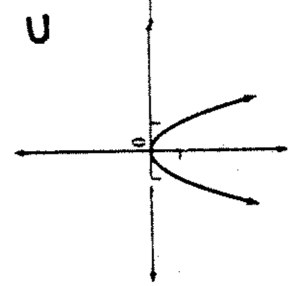
S



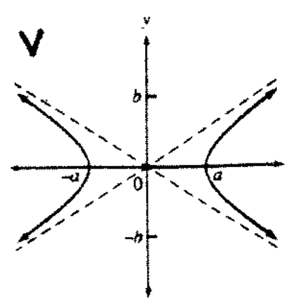
T



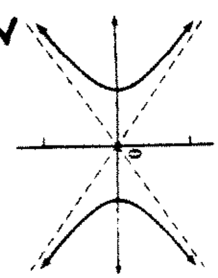
U



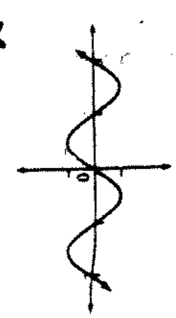
V



W



X



Y