Chp 3

3.1 Matrices

matrix ⇒

Vocab entry (or element) ⇒

square matrix ⇒

order of a matrix ⇒ $m \times n$ (m = # rows, n = # columns)

ex $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & -2 \\ 6 & 1 & 5 \end{bmatrix}$ is a 3×3 square matrix

equal ⇒

zero matrix ⇒

transpose ⇒ A^T (notation)

Ex 1 Given $A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 0 & 7 \end{bmatrix}$

(a) what size is A?

(b) what is a_{24}? a_{31}?

3.1 (cont)

Ex1 (cont)

(c) write zero matrix same size as A.

(d) what is A^T?

(e) Is A square?

(f) write $-A$.

Matrix addition / subtraction

$A \pm B \Rightarrow$

scalar multiplication

$cA \Rightarrow$

3.1 (cont)

Ex 2 Given $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 4 & 2 & 1 & 5 \\ -1 & 0 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 7 & 1 \\ 0 & 0 & -5 & 3 \\ 1 & 4 & -1 & 2 \end{bmatrix}$

and $C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 0 & 3 \\ 4 & 5 & 1 \end{bmatrix}$

(a) Find $2A + B$

(b) Find $A - 3C^T$

3.1 (cont)

Ex 3 A poll of 3320 people revealed that of the respondents that were registered Republicans, 843 approved of the president's job performance, 426 did not, and 751 had no opinion. Of the registered Democrats, 257 approved of the president's job performance, 451 did not, 92 had no opinion. Of those registered as Independents, 135 approved, 127 did not approve, and 38 had no opinion. Of the remaining respondents who were not registered, 92 approved, 64 did not approve, and 44 had no opinion. Represent this data in a 3×4 matrix.

3.2 Matrix Multiplication

Product of Two matrices

Given A $(m \times n)$ and $B(n \times p)$, AB is an $m \times p$ matrix with the ij entry given by

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$$

(i.e. the product/sum of the i^{th} row of A w/ the j^{th} column of B)

Identity matrix (I)

★ closest thing we have to a "multiplicative identity"

- always square
- has 1 in diagonal entries and 0 everywhere else
- $IA = AI = A$ (for any square matrix, same size as I)

Ex1 Given $A = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$

Find AB, and BA, if possible.

3.2 (cont)

Ex 2 Given $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 0 & 0 & -4 \end{bmatrix}$, find A^3.

Ex 3 Is $(AA^T)^T = A^T A$?

3.2 (cont)

Ex 4 Is $x=2, y=-2, z=1$ as solution for

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & -2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix} ?$$

3.3 Solving Systems of Equations

Elementary Row Operations

1. switch two rows
2. multiply any row by a nonzero constant
3. replace a row w/ the sum of a nonzero multiple of one row and a nonzero multiple of another row

Ex 1 (Augmented Matrix)

Solve

$$\begin{aligned} x+2y-z&=3 \\ 2x+5y-2z&=7 \\ -x+y+5z&=-12 \end{aligned}$$

3.3 (cont)

Ex 2 Solve (using an augmented matrix).

$$2x+3y+4z=2$$
$$x+2y+2z=1$$
$$x+y+2z=2$$

3.3 (cont)

Ex 3 Solve

$$\begin{aligned} x - y + z &= 3 \\ 3x \quad + 2z &= 7 \\ x - 4y + 2z &= 5 \end{aligned}$$

3.4 Inverse of a Square Matrix; Matrix Eqns

Defn Inverse Matrix

Two square matrices, A and B, are called inverses if $AB = I = BA$.

Notation: $B = A^{-1}$.

Ex 1 Are A + B inverses, if

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 3 \\ 4 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7/2 & 3/2 & -1 \\ -11/2 & -5/2 & 2 \\ -3 & -1 & 1 \end{bmatrix}$$

3.4 (cont)

Ex 2 Find A^{-1}, if it exists.

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

provided $ad-bc \neq 0$.

(If $ad-bc = 0$, then A^{-1} does not exist.)

3.4 (cont)

Ex 3 Find A^{-1} if $A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 4 & -3 \\ 1 & -2 & 1 \end{bmatrix}$

Ex 4 Solve

$$\begin{aligned} -x + z &= 3 \\ x + 4y - 3z &= -1 \\ x - 2y + z &= 5 \end{aligned}$$

using A^{-1} (from above).

3.4 (cont)

Determinants (only for square matrices)

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$\boxed{\text{if } \det(A) = 0,\ A^{-1} \text{ dne!}}$

Ex 5 Evaluate $\begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix}$. Does A^{-1} exist if $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$?