Math 2200
HW #8 Key (for graded problems)

4.4 #2, 8, 20, 24, 29

(2) Convert to binary notation.
32768 16384 8192 4096 2048 1024 512 256 128 64 32 16 8 4 2 1

(a) $321 = 1(256) + 1(64) + 1(1) = 101000001_2$

$321 - 256 = 65$

(b) $1023 = 1111111111_2$ (one fewer than 1024)

(c) $100,632 = 1(65536) + 1(32768) + 1(2048) + 1(256) + 1(16) + 1(8)$
$= 11000100100011000_2$

$100632 - 65536 = 35096$
$35096 - 32768 = 2328$
$2328 - 2048 = 280$
$280 - 256 = 24$

(8) Convert to hexadecimal.

(a) $1111\ 0111 = F7_{16}$

(b) $1010\ 1010\ 1010 = AAA_{16}$

(c) $111\ 0111\ 0111\ 0111$
$= 7777_{16}$

(24) Use Euclidean Algorithm to find gcd.

(a) gcd(1,5)
$5 = 5(1) \Rightarrow \gcd = 1$

(b) gcd(100,101)
$101 = 1(100) + 1$
$100 = 100(1) \Rightarrow \gcd = 1$

(c) gcd(123,277)
$277 = 2(123) + 31$
$123 = 3(31) + 30$
$31 = 1(30) + 1$
$30 = 30(1)$
$\Rightarrow \gcd = 1$

(d) gcd(1529,14039)
$14039 = 9(1529) + 278$
$1529 = 5(278) + 139$
$278 = 2(139)$
$\Rightarrow \gcd = 139$

(24) (cont)

(e) gcd(1529, 14038)

$14038 = 9(1529) + 277$

$1529 = 5(277) + 144$

$277 = 1(144) + 133$

$144 = 1(133) + 11$

$133 = 12(11) + 1$

$11 = 11(1) \Rightarrow \gcd = 1$

(f) gcd(11111, 111111)

$111111 = 10(11111) + 1$

$11111 = 11111(1) \Rightarrow \gcd = 1$

(29) Show that $n \in \mathbb{Z}^+$ is divisible by 3

$\Leftrightarrow$ sum of digits of n is divisible by 3.

pf Assume n has m digits, $a_0, a_1, \dots, a_{m-1}$ $\ni$ n can be written as $n = a_{m-1}10^{m-1} + a_{m-2}10^{m-2} + \dots + a_2 10^2 + a_1(10) + a_0$.

$a_i \in \{0,1,2,\dots,9\}$ $\forall i = 0, \dots, m-1$

Then sum of digits is $S = a_{m-1} + a_{m-2} + \dots + a_2 + a_1 + a_0$

① If $3 \mid n$, then

$n = 3d$ for some $d \in \mathbb{Z}^+$.

but 3 does not divide 10 or any power of 10

$\Rightarrow$ each term of n contains a factor of 3

$\Rightarrow$ a_i has a factor of 3 $\forall i = 0, \dots, m-1$.

$\Rightarrow 3 \mid S$ //

② If $3 \mid S$, then 3 divides every term a_i of S

$\Rightarrow 3 \mid n$. //

3.2

(20) Find $11^{644} \bmod 645$.

$b = 11 \quad m = 645$

base 2 values: 512 256 128 64 32 16 8 4 2 1

$644 = 1(512) + 0(256) + 1(128) + 1(4) =$

1010000100_2

$644 - 512 = 132$

$p = 11 \bmod 645 = 11$

$x = 1$

i	a_i	$x = (xp) \bmod 645$	$p = p^2 \pmod{645}$
			121
0	0		$121^2 \pmod{645} = 451$
1	0		$451^2 \pmod{645} = 226$
2	1	451	$226^2 \pmod{645} = 121$
3	0		$121^2 = 451 \bmod 645$
4	0		$451^2 = 226 \bmod 645$
5	0		$226^2 = 121 \bmod 645$
6	0		$121^2 = 451$
7	1	$451(121) = 391 \bmod 645$	$451^2 = 226$
8	0		
9	1	$391(226) = 1 \bmod 645$	