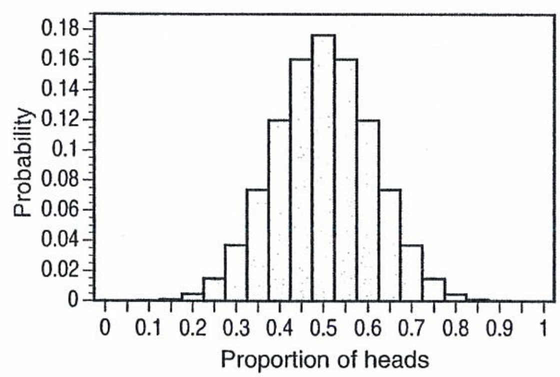
Chapter 10 Solutions

10.1. In the long run, of a large number of Texas Hold'em games in which you hold a pair, the fraction in which you can make four of a kind will be about 88/1000 (or 11/125). It *does not* mean that exactly 88 out of 1000 such hands would yield four of a kind; that would mean, for example, that if you've been dealt 999 such hands and only had four of a kind 87 times, then you could count on getting four of a kind the next time you held a pair.

10.2. **(a)** An impossible event has probability 0. **(b)** A certain event has probability 1. **(c)** An event with probability 0.01 would be called "very unlikely." **(d)** This event has probability 0.6 (or 0.99, but "more often than not" is a rather weak description of an event with probability 0.99).

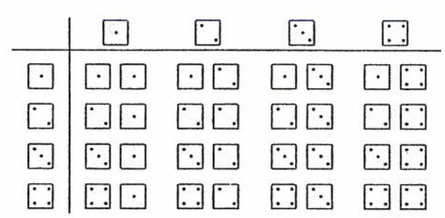
10.3. **(a)** There are 4 zeros among the first 50, for a proportion of 0.08. **(b)** Answers will vary, but more than 99% of all students should get between 7 and 33 heads out of 200 flips.

10.4. **(a)** In almost 99% of all simulations, there will be between 5 and 15 heads, so the sample proportion will be between 0.25 and 0.75. **(b)** Shown on the right is the theoretical histogram; a stemplot of 25 proportions will have roughly that shape.



10.5. **(a)** $S = \{\text{male, female}\}$. **(b)** $S = \{$All numbers between __ and __ inches$\}$. (Choices of upper and lower limits will vary.) **(c)** $S = \{$all numbers greater than or equal to 0$\}$, or $S = \{0, 0.01, 0.02, 0.03, \ldots\}$. **(d)** $S = \{\text{A, B, C, D, F}\}$ (students might also include "+" and "–").

10.6. **(a)** The table on the right illustrates the 16 possible pair combinations in the sample space. **(b)** Each of the 16 outcomes has probability 1/16.



10.7. For the sample space, add 1 to each pair-total in the table shown in the previous solution: $S = \{3, 4, 5, 6, 7, 8, 9\}$. As all faces are equally likely and the dice are independent, each of the 16 possible pairings is equally likely, so (for example) the probability of a total of 5 is 3/16, because 3 pairings add to 4 (and then we add 1). The complete set of probabilities is shown in the table on the right.

Total	Probability
3	1/16
4	2/16
5	3/16
6	4/16
7	3/16
8	2/16
9	1/16

10.8. **(a)** 35% (20% + 15%) are currently undergraduates. This makes use of Rule 3, because (assuming there are no double majors) "undergraduate students in business" and "undergraduate students in other fields" have no students in common. **(b)** 80% (100% – 20%) are not undergraduate business students. This makes use of Rule 4.

10.9. **(a)** Event B specifically rules out obese subjects, so there is no overlap with event A. **(b)** A or B is the event "The person chosen is overweight or obese." $P(A \text{ or } B) = P(A) + P(B) = 0.32 + 0.34 = 0.66$. **(c)** $P(C) = 1 - P(A \text{ or } B) = 0.34$.

10.10. **(a)** The given probabilities have sum 0.91, so $P(\text{other language}) = 0.09$. **(b)** $P(\text{not English}) = 1 - 0.63 = 0.37$. (Or, add the other three probabilities.) **(c)** $P(\text{neither English nor French}) = 0.06 + 0.09 = 0.15$. (Or, subtract $0.63 + 0.22$ from 1.)

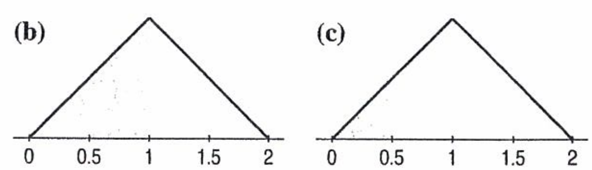
10.11. Model 1: Not legitimate (probabilities have sum $\frac{6}{7}$). Model 2: Legitimate. Model 3: Not legitimate (probabilities have sum $\frac{7}{6}$). Model 4: Not legitimate (probabilities cannot be more than 1).

10.12. **(a)** $A = \{7, 8, 9\}$, so $P(A) = 0.058 + 0.051 + 0.046 = 0.155$. **(b)** $B = \{1, 3, 5, 7, 9\}$, so $P(B) = 0.301 + 0.125 + 0.079 + 0.058 + 0.046 = 0.609$. **(c)** A or $B = \{1, 3, 5, 7, 8, 9\}$, so $P(A \text{ or } B) = 0.301 + 0.125 + 0.079 + 0.058 + 0.051 + 0.046 = 0.660$. This is different from $P(A) + P(B)$ because A and B are not disjoint.

10.13. **(a)** The eight probabilities have sum 1 (and account for all possible responses to the question). **(b)** "$X < 7$" means "the subject worked out fewer than 7 days in the past week," or "the subject did not work out every day in the past week." $P(X < 7) = 1 - 0.02 = 0.98$ (or, add the first 7 probabilities). **(c)** "Worked out at least once" is "$X > 0$." $P(X > 0) = 1 - 0.68 = 0.32$ (or, add the last 7 probabilities).

10.14. **(a)** $P(Y \leq 0.4) = 0.4$. **(b)** $P(Y < 0.4) = 0.4$. **(c)** $P(0.3 \leq Y \leq 0.5) = 0.2$.

10.15. **(a)** The area of a triangle is $\frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$. **(b)** $P(X < 1) = 0.5$. **(c)** $P(X < 0.5) = 0.125$.



10.16. **(a)** The event is $\{X \geq 10\}$. **(b)** $P(X \geq 10) = P\left(\frac{X-6.8}{1.6} \geq \frac{10-6.8}{1.6}\right) = P(Z \geq 2) \doteq 0.025$ (using the 68–95–99.7 rule) or 0.0228 (using Table A).

10.17. **(a)** $X \geq 3$ means the student's grade is either an A or a B. $P(X \geq 3) = 0.42 + 0.26 = 0.68$. **(b)** "Poorer than C" means either D ($X = 1$) or F ($X = 0$), so we want $P(X < 2) = P(X \leq 1) = 0.02 + 0.10 = 0.12$.

10.18. **(a)** $Y \geq 8$ means the student runs the mile in 8 minutes or more. $P(Y \geq 8) = P\left(\frac{Y-7.11}{0.74} \geq \frac{8-7.11}{0.74}\right) = P(Z \geq 1.2) \doteq 0.1151$ (using Table A). **(b)** "The student could run a mile in less than 6 minutes" is the event $Y < 6$. $P(Y < 6) = P\left(\frac{Y-7.11}{0.74} < \frac{6-7.11}{0.74}\right) = P(Z < -1.5) \doteq 0.0668$ (using Table A).

10.19. **(b)** A personal probability might take into account specific information about one's own driving habits, or about the kind of traffic one usually drives in. **(c)** Most people believe that they are better-than-average drivers (whether or not they have any evidence to support that belief).

10.20. **(a)** If Joe says $P(\text{Duke wins}) = 0.2$, then he believes $P(\text{Clemson wins}) = 0.1$ and $P(\text{North Carolina wins}) = 0.4$. **(b)** Joe's probabilities for Duke, Clemson, and North Carolina add up to 0.7, so that leaves probability 0.3 for all other teams.

10.21. (a) Probabilities express the *approximate* fraction of occurrences out of many trials.

10.22. (b) The set $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ lists all possible counts.

10.23. (b) This is a discrete (but not equally likely) model.

10.24. (b) The other probabilities add to 0.96, so this must be 0.04.

10.25. (c) $P(\text{type O or type B}) = P(\text{type O}) + P(\text{type B}) = 0.45 + 0.11 = 0.56$.

10.26. (a) $P(\text{not type O}) = 1 - P(\text{type O}) = 1 - 0.45 = 0.55$.

10.27. (b) There are 10 equally likely possibilities, so $P(\text{zero}) = 1/10$.

10.28. (c) "7 or greater" means 7, 8, or 9—3 of the 10 possibilities.

10.29. (a) 80% have 0, 1, or 2 cars; the rest have more than 2.

10.30. (c) $Y > 1$ standardizes to $Z > 2.56$, for which Table A gives 0.0052.

10.31. **(a)** There are sixteen possible outcomes:

{ HHHH, HHHM, HHMH, HMHH, MHHH, HHMM, HMHM, HMMH,
MHHM, MHMH, MMHH, HMMM, MHMM, MMHM, MMMH, MMMM }

(b) The sample space is {0,1,2,3,4}.

10.32. **(a)** Legitimate. **(b)** Legitimate (even if the deck of cards is not!). **(c)** Not legitimate (the total is more than 1).

10.33. **(a)** The given probabilities have sum 0.72, so this probability must be 0.28. **(b)** P(at least a high school education) $= 1 - P$(has not finished HS) $= 1 - 0.13 = 0.87$. (Or, add the other three probabilities.)

10.34. In computing the probabilities, we have dropped the trailing zeros from the land area figures. **(a)** P(area is forested) $= \frac{4176}{9094} \doteq 0.4592$. **(b)** P(area is not forested) $\doteq 1 - 0.4592 = 0.5408$.

10.35. **(a)** All probabilities are between 0 and 1, and they add to 1. (We must assume that no one takes more than one language.) **(b)** The probability that a student is studying a language other than English is $0.41 = 1 - 0.59$ (or add all the other probabilities). **(c)** This probability is $0.38 = 0.26 + 0.09 + 0.03$.

10.36. **(a)** The given probabilities add to 0.90, so other colors must account for the remaining 0.10. **(b)** P(Silver or white) $= 0.18 + 0.19 = 0.37$, so P(neither silver nor white) $= 1 - 0.37 = 0.63$.

10.37. Of the seven cards, there are three 9's, two red 9's, and two 7's. **(a)** P(draw a 9) $= \frac{3}{7}$. **(b)** P(draw a red 9) $= \frac{2}{7}$. **(c)** P(don't draw a 7) $= 1 - P$(draw a 7) $= 1 - \frac{2}{7} = \frac{5}{7}$.

10.38. The probabilities of 2, 3, 4, and 5 are unchanged (1/6), so P(⚀ or ⚅) must still be 1/3. If P(⚅) $= 0.2$, then P(⚀) $= \frac{1}{3} - 0.2 = 0.1\overline{3}$ (or $\frac{2}{15}$).

Face	⚀	⚁	⚂	⚃	⚄	⚅
Probability	$0.1\overline{3}$	1/6	1/6	1/6	1/6	0.2

10.39. Each of the 90 guests has probability 1/90 of winning the prize. The probability that the winner is a woman is the sum of 1/90 42 times, one for each woman. The probability is $42/90 = 0.4\overline{6}$.

10.40. **(a)** It is legitimate because every person must fall into exactly one category, the probabilities are all between 0 and 1, and they add up to 1. **(b)** $0.149 = 0.001 + 0.006 + 0.139 + 0.003$ is the probability that a randomly chosen American is Hispanic. **(c)** $0.326 = 1 - 0.674$ is the probability that a randomly chosen American is not a non-Hispanic white.

10.41. **(a)** It is legitimate because every person must fall into exactly one category, the probabilities are all between 0 and 1, and they add up to 1. **(b)** P(19-year-old with his/her own place) $= 0.04$. **(c)** P(19-year-old) $= 0.18$—the sum of the numbers in the first column. **(d)** P(lives in his/her own place) $= 0.42$—the sum of the numbers in the second row.

10.42. **(a)** A corresponds to the outcomes in the first column and the second row. **(b)** Adding up those 6 outcomes gives $P(A) = 0.56$. This is different from the sum of the probabilities in (c) and (d) because that sum counts the overlap (0.04) twice.

10.43. **(a)** P(21 years old or older) $= 0.56$—the sum of the third and fourth columns. **(b)** P(does not live with his/her parents) $= 0.54$—the sum of the second and third rows (or one minus the sum of the first row).

10.44. **(a)** X is discrete, because it has a finite sample space. **(b)** "At least one nonword error" is the event $\{X \geq 1\}$ (or $\{X > 0\}$). $P(X \geq 1) = 1 - P(X = 0) = 0.9$. **(c)** $\{X \leq 2\}$ is "no more than two nonword errors," or "fewer than three nonword errors." $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1 + 0.2 + 0.3 = 0.6$. $P(X < 2) = P(X = 0) + P(X = 1) = 0.1 + 0.2 = 0.3$.

Note: *The more precise notation* $\{X \geq 1\}$ *is clearer in print than just* $X \geq 1$. *We recommend that you* not *use the precise notation for students at this level.*

10.45. **(a)** All 9 digits are equally likely, so each has probability 1/9:

Value of W	1	2	3	4	5	6	7	8	9
Probability	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

(b) $P(W \geq 6) = \frac{4}{9} \doteq 0.444$—twice as big as the Benford's law probability.

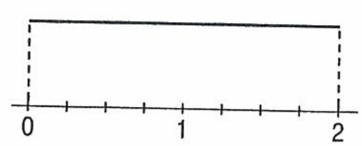
10.46. **(a)** There are 10 pairs. Just using initials: {(A, D), (A, M), (A, S), (A, R), (D, M), (D, S), (D, R), (M, S), (M, R), (S, R)}. **(b)** Each has probability $1/10 = 10\%$. **(c)** Mei-Ling is chosen in 4 of the 10 possible outcomes: $4/10 = 40\%$. **(d)** There are 3 pairs with neither Sam nor Roberto, so the probability is 3/10.

10.47. **(a)** BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG. Each has probability 1/8. **(b)** Three of the eight arrangements have two (and only two) girls, so $P(X = 2) = 3/8 = 0.375$. **(c)** See table at right.

Value of X	0	1	2	3
Probability	1/8	3/8	3/8	1/8

10.48. The possible values of Y are $1, 2, 3, \ldots, 12$, each with probability 1/12. Aside from drawing a diagram showing all the possible combinations, one can reason that the first (regular) die is equally likely to show any number from 1 through 6. Half of the time, the second roll shows 0, and the other half it shows 6. Each possible outcome therefore has probability $\frac{1}{6} \cdot \frac{1}{2}$.

10.49. **(a)** This is a continuous random variable because the set of possible values is an interval. **(b)** The height should be $\frac{1}{2}$ because the area under the curve must be 1. The density curve is at right. **(c)** $P(Y \leq 1) = \frac{1}{2}$.



10.50. For these probabilities, compute the areas of the appropriate rectangle under the density shown above (Exercise 10.49). **(a)** $P(0.5 < Y < 1.3) = 0.4$. **(b)** $P(Y \geq 0.8) = 0.6$.

10.51. **(a)** $P(0.52 \leq V \leq 0.60) = P\left(\frac{0.52-0.56}{0.019} \leq \frac{V-0.56}{0.019} \leq \frac{0.60-0.56}{0.019}\right) = P(-2.11 \leq Z \leq 2.11) = 0.9826 - 0.0174 = 0.9652$. **(b)** $P(V \geq 0.72) = P\left(\frac{V-0.56}{0.019} \geq \frac{0.72-0.56}{0.019}\right) = P(Z \geq 8.42)$; this is basically 0.

10.52. $P(8.9 \le \bar{x} \le 9.1) = P\left(\frac{8.9-9}{0.075} \le \frac{\bar{x}-9}{0.075} \le \frac{9.1-9}{0.075}\right) \doteq P(-1.33 \le Z \le 1.33) =$ $0.9082 - 0.0918 = 0.8164$. (Software give 0.8176.)

10.53. (a) Because there are 10,000 equally likely four-digit numbers (0000 through 9999), the probability of an exact match is $\frac{1}{10{,}000}$. (b) There is a total of $24 = 4 \cdot 3 \cdot 2 \cdot 1$ arrangements of the four digits 5, 9, 7, and 4 (there are four choices for the first digit, three for the second, two for the third), so the probability of a match in any order is $\frac{24}{10{,}000}$.

10.54. Note that in this experiment, factors other than the nickel's characteristics might affect the outcome. For example, if the surface used is not quite level, there will be a tendency for the nickel to fall in the "downhill" direction.

10.55. (a) – (c) Results will vary, but after n tosses, the distribution of the proportion $\hat{p}$ is approximately Normal with mean 0.5 and standard deviation $1/(2\sqrt{n})$, while the distribution of the count of heads is approximately Normal with mean $0.5n$ and standard deviation $\sqrt{n}/2$, so using the 68–95–99.7 rule, we have the results shown in the table on the right. Note that the range for $\hat{p}$ gets narrower, while the range for the count gets wider.

n	99.7% Range for $\hat{p}$	99.7% Range for count
40	0.5 ± 0.237	20 ± 9.5
120	0.5 ± 0.137	60 ± 16.4
240	0.5 ± 0.097	120 ± 23.2
480	0.5 ± 0.068	240 ± 32.9

10.56. (a) Most answers will be between 35% and 65%. (b) Based on 10,000 simulated trials—more than students are expected to do—there is about an 80% chance of having a longest run of four or more (i.e., either making or missing four shots in a row), a 54% chance of getting five or more, a 31% chance of getting six or more, and a 16% chance of getting seven or more. The average ("expected") longest run length is about six.

10.57. (a) With $n = 20$, nearly all answers will be 0.40 or greater. With $n = 80$, nearly all answers will be between 0.50 and 0.80. With $n = 320$, nearly all answers will be between 0.58 and 0.72.