# Chapter 11 Solutions

11.1. Both 283 and 311 pushes per minute are statistics (related to one sample: the subjects with placebo, and the same subjects with caffeine).

11.2. 41% and 37% are parameters (related to the population of all registered voters in Florida); 33% is a statistic (related to the sample of registered voters among those called).

11.3. Both 82% and 18% are statistics, related to those projectile points found at the North Carolina site. (This assumes that we can view those points as a sample from some population—either the population of all points buried at that site, or all points buried in North America.)

**11.4.** Sketches will vary; one result is shown on the right.
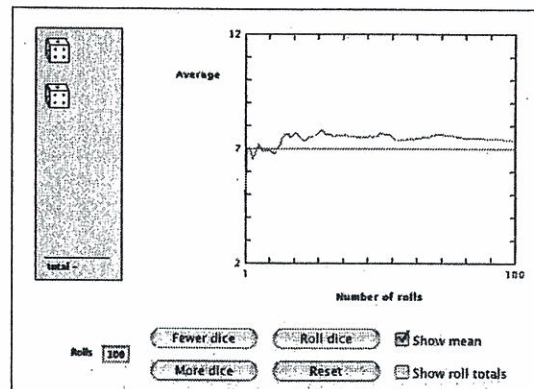
11.5. Although the probability of having to pay for a total loss for 1 or more of the 12 policies is very small, if this were to happen, it would be financially disastrous. On the other hand, for thousands of policies, the law of large numbers says that the average claim on many policies will be close to the mean, so the insurance company can be assured that the premiums they collect will (almost certainly) cover the claims.

11.6. (a) The population is the 12,000 students; the population distribution (Normal with mean 7.11 minutes and standard deviation 0.74 minute) describes the time it takes a randomly selected individual to run a mile. **(b)** The sampling distribution (Normal with mean 7.11 minutes and standard deviation 0.074 minute) describes the average mile-time for 100 randomly selected students.

11.7. (a) $\mu = 694/10 = 69.4$. **(b)** The table on the following page shows the results for line 116. Note that we need to choose 5 digits, because the digit 4 appears twice. (When choosing an SRS, no student should be chosen more than once.) **(c)** The results for the other lines are in the table; the histogram is shown on the right on the following page. (Students might choose different intervals than those shown here.) The center of the histogram is a bit lower than 69.4 (it is closer to about 67), but for a small group of $\bar{x}$-values, we should not expect the center to be in exactly the right place.

    Note: *You might consider having students choose different samples from those prescribed*

in this exercise, and then pooling the results for the whole class. With more values of $\bar{x}$, a better picture of the sampling distribution begins to develop.

Line	Digits	Scores	$\bar{x}$
116	14459	62 + 72 + 73 + 62 = 269	67.25
117	3816	58 + 74 + 62 + 65 = 259	64.75
118	7319	66 + 58 + 62 + 62 = 248	62
119	95857	62 + 73 + 74 + 66 = 275	68.75
120	3547	58 + 73 + 72 + 66 = 269	67.25
121	7148	66 + 62 + 72 + 74 = 274	68.5
122	1387	62 + 58 + 74 + 66 = 260	65
123	54580	73 + 72 + 74 + 82 = 301	75.25
124	7103	66 + 62 + 82 + 58 = 268	67
125	9674	62 + 65 + 66 + 72 = 265	66.25
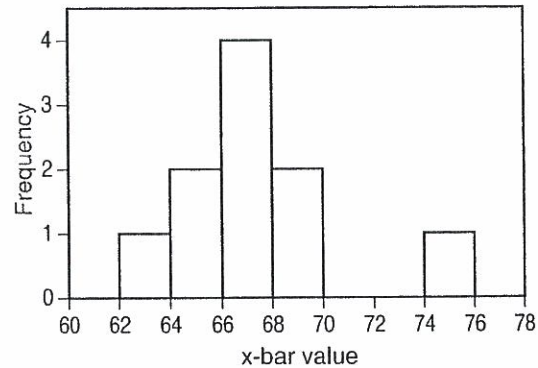
11.8. (a) $\bar{x}$ is not systematically higher than or lower than μ; that is, it has no particular tendency to underestimate or overestimate μ. **(b)** With large samples, $\bar{x}$ is more likely to be close to μ, because with a larger sample comes more information (and therefore less uncertainty).

11.9. (a) The sampling distribution of $\bar{x}$ is $N(188, 41/\sqrt{100}) = N(188 \text{ mg/dl}, 4.1 \text{ mg/dl})$. Therefore, $P(185 < \bar{x} < 191) \doteq P(-0.73 < Z < 0.73) \doteq 0.5346$. **(b)** With $n = 1000$, the sample mean has the $N(188 \text{ mg/dl}, 1.2965 \text{ mg/dl})$ distribution, so $P(185 < \bar{x} < 191) \doteq P(-2.31 < Z < 2.31) \doteq 0.9792$.

11.10. (a) $\sigma/\sqrt{3} \doteq 5.7735$ mg. **(b)** Solve $\sigma/\sqrt{n} = 5$: $\sqrt{n} = 2$, so $n = 4$. The average of several measurements is more likely than a single measurement to be close to the mean.

11.11. No: the histogram of the sample values will look like the population distribution, whatever it might happen to be. (For example, if we roll a fair die many times, the histogram of sample values should look relatively flat—probability close to 1/6 for each value 1, 2, 3, 4, 5, and 6.) The central limit theorem says that the histogram of *sample means* (from many large samples) will look more and more Normal.

11.12. (a) $\mu_{\bar{x}} = 0.5$ and $\sigma_{\bar{x}} = \sigma/\sqrt{50} = 0.7/\sqrt{50} \doteq 0.09899$. **(b)** Because this distribution is only approximately Normal, it would be quite reasonable to use the 68–95–99.7 rule to give a rough estimate: 0.6 is about one standard deviation above the mean, so the probability should be about 0.16 (half of the 32% that falls outside ± 1 standard deviation). Alternatively, $P(\bar{x} > 0.6) \doteq P\left(Z > \frac{0.6-0.5}{0.09899}\right) = P(Z > 1.01) = 0.1562$.

11.13. STATE: What is the probability that the average loss will be no greater than $275? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that, in spite of the skewness of the population distribution, the average loss among 10,000 policies will be approximately $N(\$250, \sigma/\sqrt{10,000}) = N(\$250, \$10)$. Because $275 is 2.5 standard deviations above the mean, the probability of seeing an average loss of no more than $275 is about 0.9938. CONCLUDE: We can be about 99.4% certain that average losses will not exceed $275 per policy.

11.14. (c) This is a proportion of the people interviewed in the sample of 60,000 households.

11.15. (b) 56% is a proportion of all registered voters (the population).

11.16. (b) The law of large numbers says that the mean from a large sample is close to the population mean. Statement (c) is also true, but is based on the central limit theorem, not on the law of large numbers.

11.17. (a) The mean of the sample means ($\bar{x}$) is the same as the population mean (μ).

11.18. (c) The standard deviation of the distribution of $\bar{x}$ is $\sigma/\sqrt{n}$.

11.19. (a) "Unbiased" means that the estimator is right "on the average."

11.20. (c) The central limit theorem says that the mean from a large sample has (approximately) a Normal distribution. Statement (a) is also true, but is based on the law of large numbers, not on the central limit theorem.

11.21. (b) For $n = 6$ women, $\bar{x}$ has a $N(266, 16/\sqrt{6}) \doteq N(266, 6.5320)$ distribution, so $P(\bar{x} > 270) \doteq P(Z > 0.61) = 0.2709$.

11.22. 1 is a parameter (the mean of the population of all conductivity measurements); 1.09 is a statistic (the mean of the 10 measurements in the sample).

11.23. Both 40.2% and 31.7% are statistics (related, respectively, to the samples of small-class and large-class black students).

11.24. In the long run, the gambler earns an average of 94.7 cents per bet. In other words, the gambler loses (and the house gains) an average of 5.3 cents for each \$1 bet.

11.25. If $\bar{x}$ is the mean number of strikes per square kilometer, then $\mu_{\bar{x}} = 6$ strikes/km^2 and $\sigma_{\bar{x}} = 2.4/\sqrt{10} \doteq 0.7589$ strikes/km^2.

11.26. (a) For the height H of an individual student, $P(69 < H < 71) \doteq$ $P\left(\frac{69-70}{2.8} < Z < \frac{71-70}{2.8}\right) = P(-0.36 < Z < 0.36) = 0.6406 - 0.3594 = 0.2812$. (With software: 0.2790.) (b) With $n = 25$, the sampling distribution is $N(70, 2.8/\sqrt{25}) = N(70, 0.56)$. (c) $P(69 < \bar{x} < 71) \doteq P\left(\frac{69-70}{0.56} < Z < \frac{71-70}{0.56}\right) = P(-1.79 < Z < 1.79) = 0.9633 - 0.0367 = 0.9266$. (With software: 0.9259.)

11.27. Let X be Shelia's measured glucose level. (a) $P(X > 140) = P(Z > 1.5) = 0.0668$. (b) If $\bar{x}$ is the mean of four measurements (assumed to be independent), then $\bar{x}$ has a $N(125, 10/\sqrt{4}) = N(125 \text{ mg/dl}, 5 \text{ mg/dl})$ distribution, and $P(\bar{x} > 140) = P(Z > 3) = 0.0013$.

11.28. (a) The mean of five untreated specimens has a standard deviation of $2.3/\sqrt{5} \doteq$ 1.0286 lb, so $P(\bar{x}_u > 50) = P(Z > \frac{50-58}{1.0286}) = P(Z > -7.78)$, which is basically 1.

(b) The mean of five treated specimens has a standard deviation of $1.6/\sqrt{5} \doteq 0.7155$ lb, so $P(\bar{x}_t > 50) = P(Z > \frac{50-30}{0.7155}) = P(Z > 27.95)$, which is basically 0. deviation is

11.29. The mean of four measurements has a $N(125 \text{ mg/dl}, 5 \text{ mg/dl})$ distribution, and $P(Z > 1.645) = 0.05$ if Z is $N(0, 1)$, so $L = 125 + 1.645 \cdot 5 = 133.225$ mg/dl.

11.30. (a) For the emissions E of a single car, $P(E > 0.3) \doteq P(Z > \frac{0.3-0.2}{0.05}) = P(Z > 2) = 0.0228$ (or 0.025, using the 68–95–99.77 rule). **(b)** The average $\bar{x}$ is Normal with mean 0.3 g/mi and standard deviation $0.05/\sqrt{25} = 0.01$ g/mi. Therefore, $P(\bar{x} > 0.3) \doteq P(Z > \frac{0.3-0.2}{0.01}) = P(Z > 10)$, which is basically 0.

11.31. (a) $\bar{x}$ is approximately $N(2.2, 1.4/\sqrt{52}) = N(2.2 \text{ accidents}, 0.1941 \text{ accidents})$.
(b) $P(\bar{x} < 2) \doteq P(Z < -1.03) = 0.1515$. (Software gives the same answer.) **(c)** Let A be the number of accidents in a year; then $\bar{x} = A/52$. $P(A < 100) = P(\bar{x} < \frac{100}{52}) \doteq P(Z < -1.43) = 0.0764$ (software: 0.0769). [Alternatively, we might use the *continuity correction* (which adjusts for the fact that counts must be whole numbers) and find $P(A < 99.5) = P(\bar{x} < \frac{99.5}{52}) = P(Z < -1.48) = 0.0694$ (software: 0.0700).]

11.32. The mean NOX level for 25 cars has a $N(0.2 \text{ g/mi}, 0.01 \text{ g/mi})$ distribution, and $P(Z > 2.326) = 0.01$ if Z is $N(0, 1)$, so $L = 0.2 + 2.326 \cdot 0.01 \doteq 0.2233$ g/mi.

11.33. STATE: What are the probabilities of an average return over 10%, or less than 5%?
PLAN: Use the central limit theorem to approximate this probability.
SOLVE: The central limit theorem says that over 40 years, $\bar{x}$ (the mean return) is approximately Normal with mean $\mu = 8.7\%$ and standard deviation $\sigma_{\bar{x}} = 20.2\%/\sqrt{40} \doteq 3.194\%$. Therefore, $P(\bar{x} > 10\%) = P(Z > 0.41) = 0.3409$, and $P(\bar{x} < 5\%) = P(Z < -1.16) = 0.1230$. (Software gives 0.3420 and 0.1233.)
CONCLUDE: There is about a 34% chance of getting average returns over 10%, and a 12% chance of getting average returns less than 5%.
Note: *We have to assume that returns in separate years are independent.*

11.34. STATE: What is the probability that the total weight of the passengers exceeds 4000 lb?
PLAN: Use the central limit theorem to approximate this probability.
SOLVE: If W is total weight, and $\bar{x} = W/19$, then the central limit theorem says that $\bar{x}$ is approximately Normal with mean 190 lb and standard deviation $35/\sqrt{19} \doteq 8.0296$ lb. Therefore,
$$P(W > 4000) = P(\bar{x} > \tfrac{4000}{19}) \doteq P(Z > \tfrac{210.5263-190}{35/\sqrt{19}}) = P(Z > 2.56) = 0.0052$$
(software: 0.0053).
CONCLUDE: There is very little chance—about 0.5%—that the total weight exceeds 4000 lb.

11.35. We need to choose n so that $2.8/\sqrt{n} = 0.5$. That means $\sqrt{n} = 5.6$, so $n = 31.36$. Because n must be a whole number, take $n = 32$.

11.36. (a) 99.7% of all observations fall within 3 standard deviations, so we want $3\sigma/\sqrt{n} = 0.5$. The standard deviation of $\bar{x}$ must therefore be $1/6 = 0.1\overline{6}$ inch. (b) We need to choose n so that $2.8/\sqrt{n} = \frac{1}{6}$. That means $\sqrt{n} = 16.8$, so $n = 282.24$. Because n must be a whole number, take $n = 283$.

11.37. On the average, Joe loses 40 cents each time he plays (that is, he spends $1 and gets back 60 cents).

11.38. (a) With $n = 14{,}000$, $\mu_{\bar{x}} = \$0.60$ and $\sigma_{\bar{x}} = \$18.96/\sqrt{14{,}000} \doteq \$0.1602$.
(b) $P(\$0.50 < \bar{x} < \$0.70) \doteq P(-0.62 < Z < 0.62) \doteq 0.4648$ (software gives 0.4674).

11.39. (a) With $n = 150{,}000$, $\mu_{\bar{x}} = \$0.40$ and $\sigma_{\bar{x}} = \$18.96/\sqrt{150{,}000} \doteq \$0.0490$.
(b) $P(\$0.30 < \bar{x} < \$0.50) \doteq P(-2.04 < Z < 2.04) \doteq 0.9586$ (software gives 0.9589).

11.40. (a) The estimate in Exercise 11.38 was 0.4648 (Table A) or 0.4674 (software), so the Normal approximation slightly underestimates the exact answer. (b) With $n = 3500$, the Normal approximation gives $P(\$0.50 < \bar{x} < \$0.70) \doteq P(-0.31 < Z < 0.31)$, which is 0.2434 (Table A) or 0.2450 (software). This is quite a bit smaller than the exact answer. (c) The probability that their average winnings fall between $0.50 and $0.70 is the same as the probability found in part (b) of the previous exercise, for which the Normal approximation gives 0.9586 (Table A) or 0.9589 (software), so the approximation differs from the exact value by only about 0.004.

11.41. The mean is 10.5 ($= 3 \times 3.5$ because a single die has a mean of 3.5). Sketches will vary, as will the number of rolls; one result is shown on the right.