# Chapter 14 Solutions

14.1. (a) The standard deviation is $\sigma/\sqrt{840} \doteq 2.0702$. **(b)** The missing number is $2\sigma/\sqrt{840} \doteq 4.1404$. **(c)** The 95% confidence interval is $\bar{x} \pm 2\sigma/\sqrt{840} \doteq 268$ to 276.

14.2. Shown below are sample output screens for (a) 10 and (b) 1000 SRSs. In 99.4% of all repetitions of part (a), students should see between 5 and 10 hits (that is, at least 5 of the 10 SRSs capture the true mean μ). Out of 1000 80% confidence intervals, nearly all students will observe between 76% and 84% capturing the mean.
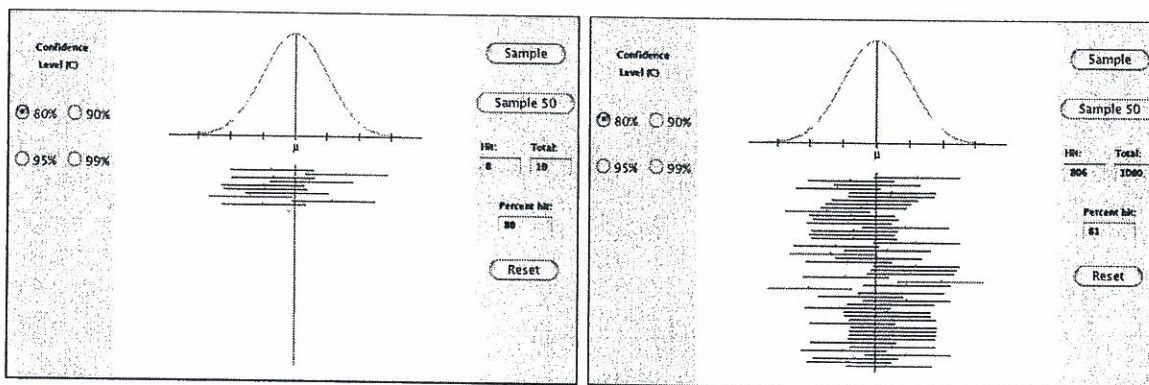
14.3. Search Table A for 0.0125 (half of the 2.5% that is *not* included in a 97.5% confidence interval). This area corresponds to $z^* = 2.24$. Software gives $z^* = 2.2414$.
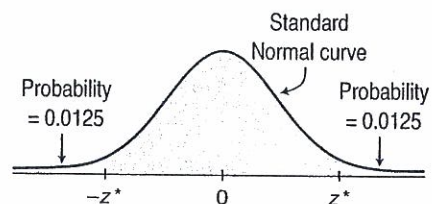
14.4. STATE: What is the true conductivity of this liquid?

PLAN: We will estimate the true conductivity μ, the mean of all measurements of its conductivity, by giving a 90% confidence interval.

SOLVE: The statement of the problem in the text suggests that the conditions for inference should be satisfied. The mean of the sample is $\bar{x} = 4.988\overline{3}$ microsiemens per centimeter (μS/cm). For 90% confidence, the critical value is $z^* = 1.645$. A 90% confidence interval for μ is therefore

$$\bar{x} \pm z^*\left(\frac{\sigma}{\sqrt{n}}\right) = 4.988\overline{3} \pm 1.645\left(\frac{0.2}{\sqrt{6}}\right) = 4.988\overline{3} \pm 0.1343 = 4.8540 \text{ to } 5.1226 \ \mu\text{S/cm}.$$

CONCLUDE: We are 90% confident that the true conductivity is between 4.8540 and 5.1226 μS/cm.

14.5. (a) The two low scores (72 and 74) are both possible outliers, but there are no other apparent deviations from Normality. **(b)** STATE: What is the mean IQ μ of all seventh-grade girls in this school district? PLAN: We will estimate μ by giving a 99% confidence interval. SOLVE: The problem states that these girls are an SRS of the population. In part (a), we saw that the scores appear to come from a Normal distribution. With $\bar{x} \doteq 105.84$, our 99% confidence interval for μ is

$$105.84 \pm 2.576 \left(\frac{15}{\sqrt{31}} \right) \doteq 105.84 \pm 6.94 = 98.90 \text{ to } 112.78 \text{ IQ points.}$$

CONCLUDE: We are 99% confident that the mean IQ of seventh-grade girls in this district is between 98.90 and 112.78.

```
 7 | 24
 7 |
 8 |
 8 | 69
 9 | 13
 9 | 68
10 | 023334
10 | 578
11 | 11222444
11 | 89
12 | 0
12 | 8
13 | 02
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14.6. (a) If $\mu = 115$, the distribution is approximately Normal with mean $\mu = 115$ and standard deviation $\sigma/\sqrt{25} = 6$. **(b)** The actual result lies out toward the high tail of the curve, while 118.6 is fairly close to the middle. If $\mu = 115$, observing a value similar to 118.6 would not be too surprising, but 125.8 is less likely, and it therefore provides some evidence that $\mu > 115$.
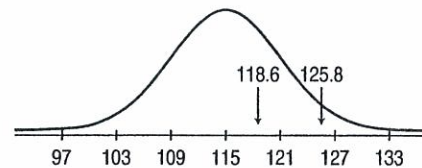

14.7. (a) If the claim is true, the sampling distribution of $\bar{x}$ is Normal with mean 5 μS/cm and standard deviation $\sigma/\sqrt{6} \doteq 0.0816$ μS/cm. **(b)** 4.98 is less than 0.25 standard deviations below the presumed mean, while 4.7 is about 3.67 standard deviations below. If $\mu = 5$, observing a value similar to 4.98 would not be too surprising, but 4.7 is less likely, and it therefore provides evidence that μ is different from 5. (Specifically, it suggests that $\mu < 5$.)
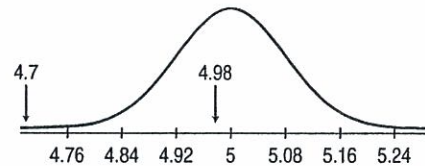

14.8. $H_0: \mu = 115$ vs. $H_a: \mu > 115$. (Because the teacher suspects that older students have a higher mean, we use a one-sided alternative.)

14.9. $H_0: \mu = 5$ vs. $H_a: \mu \neq 5$. (We are concerned about deviation from 5 in either direction, so we use a two-sided alternative.)

14.10. $H_0: \mu = 50$ vs. $H_a: \mu < 50$. (Because the professor suspects that this TA's students will have lower scores, we use a one-sided alternative.)

14.11. $H_0: \mu = 64.2$ in vs. $H_a: \mu \neq 64.2$ in. (We have no prior suspicion that this group is taller or shorter than the national mean, so we use a two-sided alternative.)

14.12. Hypotheses should be stated in terms of μ, not $\bar{x}$.

   Note: *Students who think about this problem a bit more might also point out that 1000 g (2.2 lb) is a dangerously low birth weight; babies smaller than this are classified as extremely low birth weight (ELBW).*

14.13. (a) With $\sigma = 1$ and $n = 10$, the standard deviation is $\sigma/\sqrt{10} \doteq 0.3162$, so when $\mu = 0$, the distribution of $\bar{x}$ is $N(0, 0.3162)$. **(b)** The P-value is
$$P = P(\bar{x} > 0.3) = P\left(Z \geq \frac{0.3 - 0}{0.3162}\right) \doteq 0.1711.$$

14.14. (a) With $\sigma = 60$ and $n = 18$, the standard deviation is $\sigma/\sqrt{18} \doteq 14.1421$, so when $\mu = 0$, the distribution of $\bar{x}$ is $N(0, 14.1421)$. **(b)** The P-value is
$$P = 2P(\bar{x} > 17) = 2P\left(Z \geq \frac{17 - 0}{14.1421}\right) \doteq 0.2302.$$

14.15. (a) The results observed in this study would rarely have occurred by chance if vitamin C were ineffective. **(b)** $P < 0.01$ means that results similar to those observed would occur less than 1% of the time if vitamin C supplements had no effect.

14.16. (a) The P-value for $\bar{x} = 118.6$ is 0.2743. This is not significant at either $\alpha = 0.05$ or $\alpha = 0.01$. **(b)** The P-value for $\bar{x} = 125.8$ is 0.0359. This is significant at $\alpha = 0.05$, but not at $\alpha = 0.01$. **(c)** If $\mu = 115$ (that is, if H_0 were true), observing a value similar to 118.6 would not be too surprising, but 125.8 is less likely, and it therefore provides some evidence that $\mu > 115$.
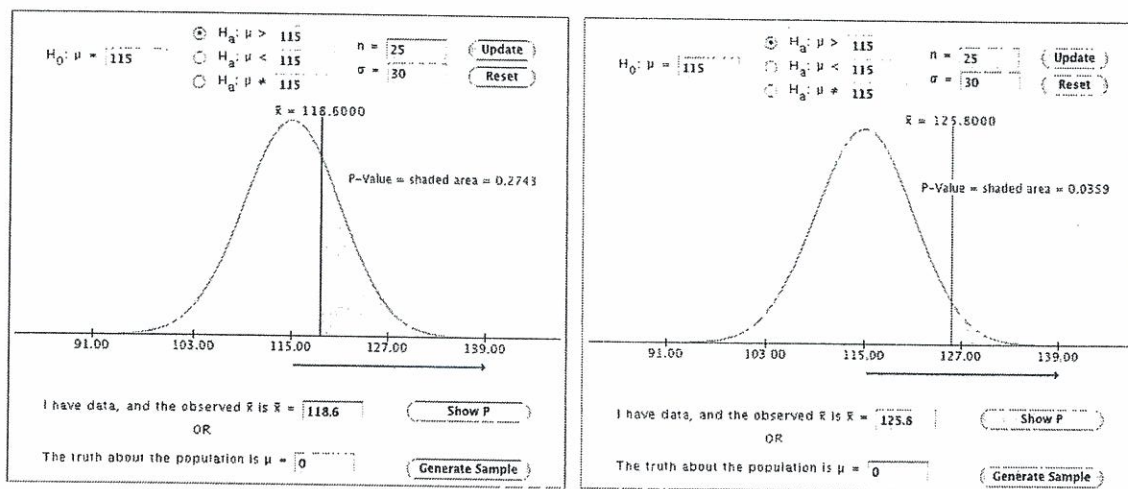
14.17. (a) The P-value for $\bar{x} = 4.98$ is 0.8104. This is not significant at either $\alpha = 0.05$ or $\alpha = 0.01$. **(b)** The P-value for $\bar{x} = 4.7$ is 0.0002. This is significant at both $\alpha = 0.05$ and $\alpha = 0.01$. **(c)** If $\mu = 5$ (that is, if H_0 were true), observing a value similar to 4.98 would not be too surprising, but 4.7 is less likely, and it therefore provides strong evidence that μ is different from 5. (Specifically, it suggests that $\mu < 5$.)
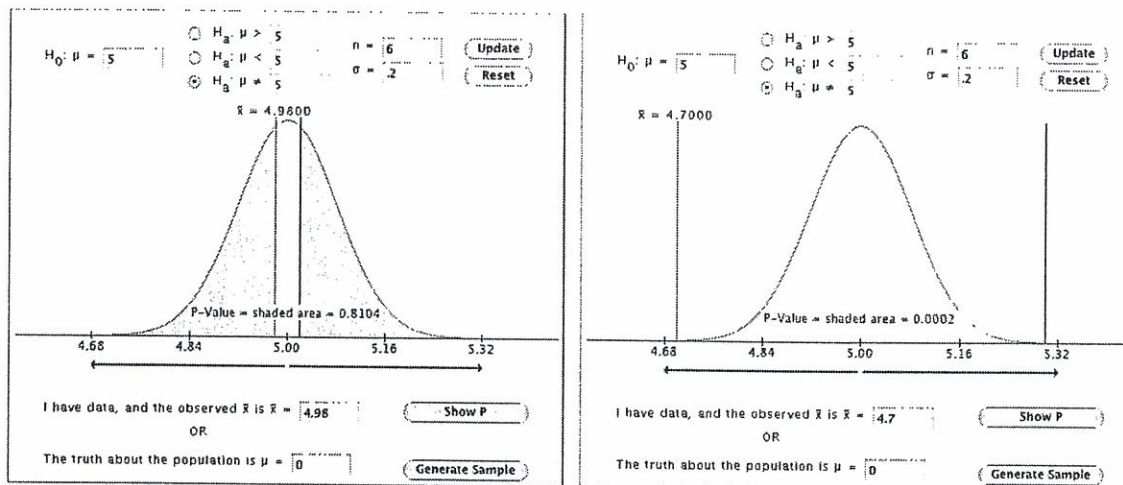
14.18. The P-values for (a) and (c) are computed in Exercises 14.13 and 14.14.
  (a) For $\bar{x} = 0.3$, $z = \frac{0.3-0}{1/\sqrt{10}} \doteq 0.95$. **(b)** For $\bar{x} = 1.02$, $z = \frac{1.02-0}{1/\sqrt{10}} \doteq 3.23$.
  (c) For $\bar{x} = 17$, $z = \frac{17-0}{60/\sqrt{18}} \doteq 1.20$.

14.19. STATE: Is there evidence that the true conductivity of the liquid is not 5?
  PLAN: Let μ be the liquid's true conductivity (the mean of all measurements of its conductivity). We test H_0: $\mu = 5$ vs. H_a: $\mu \neq 5$; we are concerned about deviation from 5 in either direction, so we use a two-sided alternative.
  SOLVE: Assume we have a Normal distribution and an SRS. We find that $\bar{x} = 4.9883$, so the test statistic is $z = \frac{4.9883-5}{0.2/\sqrt{6}} \doteq -0.14$, and the P-value is $P = 2P(Z < -0.14) \doteq 0.8886$.
  CONCLUDE: This sample gives little reason to doubt that the true conductivity is 5.

14.20. STATE: Does the use of fancy type fonts slow down the reading of text?
  PLAN: Let μ be the mean reading time for Gigi. We test H_0: $\mu = 22$ sec vs. H_a: $\mu > 22$ sec; the alternative is one-sided because we expect that the fancy font will increase reading times.
  SOLVE: Assume we have a Normal distribution and an SRS. We find that $\bar{x} = 27.088$ sec, so the test statistic is $z = \frac{27.088-22}{6/\sqrt{25}} \doteq 4.24$, and the P-value is $P = P(Z > 4.24) \doteq 0$.
  CONCLUDE: A mean reading time as large as 27.088 seconds would almost never occur if Gigi did not affect reading time ($\mu = 22$ sec). This is very strong evidence (significant at $\alpha = 0.01$, or much smaller) that it takes more than 22 seconds to read text printed in Gigi.

14.21. For a one-sided test, $z = 1.776$ is significant at 5% but not at 1%. To see this, either compute the P-value for $z = 1.776$ (which is $P = 0.0379$), or note that 1.776 falls between the 5% and 1% critical values from Table C (1.645 and 2.326, respectively).

14.22. For a two-sided test, $z = 1.776$ is not significant at either level. To see this, either compute the P-value for $z = 1.776$ (which is $P = 0.0757$), or note that 1.776 does not fall beyond the 2.5% and 0.5% critical values from Table C (1.960 and 2.576, respectively).

14.23. (a) $z = \frac{0.4365-0.5}{0.2887/\sqrt{100}} \doteq -2.20$. **(b)** This result is significant at the 5% level because $z < -1.960$. **(c)** It is not significant at 1% because $z \geq -2.576$. **(d)** This value of z is between 2.054 and 2.326, so the P-value is between 0.02 and 0.04 (because the alternative is two-sided).

14.24. (b) Table C (or software) shows that $z^* = 2.054$ for 96% confidence.

14.25. (b) To be significant at level α, we need $P < \alpha$.

14.26. (c) The P-value for $z = 2.433$ is 0.0075 (assuming that the difference is in the correct direction; that is, assuming that the alternative hypothesis was $\mu > \mu_0$).

14.27. (a) The standard deviation is $\sigma/\sqrt{n} = \sigma/\sqrt{3} \doteq 0.000577$ gram, so the margin of error is $1.96\sigma/\sqrt{3} = (1.96)(0.000577) \doteq 0.00113$.

14.28. (b) Greater confidence—with the same sample size—requires a larger margin of error. (Specifically, the 99% confidence interval would be $\frac{2.576}{1.96} \doteq 1.31$ times larger than the 95% interval.)

14.29. (c) $z = \frac{3.414-3.41}{0.001/\sqrt{3}} \doteq 6.928$.

14.30. (c) The margin of error is $2.576\sigma/\sqrt{8} = (2.576)(0.000354) \doteq 0.00091$.

14.31. (a) The null hypothesis states that μ takes on the "default" value, 18 seconds.

14.32. (b) The researcher believes that loud noises will decrease the completion time, so the alternative is one-sided.

14.33. (c) A P-value gives the likelihood of observing results at least as extreme as those in our data, when we assume H_0 is true.

14.34. (a) The margin of error for 99% confidence is $2.576\left(\frac{65}{\sqrt{269}}\right) \doteq 10.2090$ minutes, so the interval is $137 \pm 10.2090 = 126.8$ to 147.2 minutes. **(b)** We need to know if this sample (i.e., the students in the class where the survey was performed) can be considered an SRS of the population of all first-year students at this university.

14.35. The margin of error for 90% confidence is $1.645\left(\frac{2.5}{\sqrt{200}}\right) \doteq 0.2908$ kg/m², so the interval is $2.35 \pm 0.2908 = 2.0592$ to 2.6408 kg/m².

14.36. The margin of error for 99% confidence is slightly smaller (because of the larger sample size). It is now $2.576\left(\dfrac{65}{\sqrt{270}}\right) \doteq 10.1901$ minutes, so the interval is $248 \pm 10.1901 = 237.8$ to 258.2 minutes, compared with 126.8 to 147.2 minutes in Exercise 14.34. This one outlier has a huge impact on the interval.

14.37. No: the interval refers to the mean BMI of all women, not to individual BMIs, which will be much more variable.

14.38. This is not quite correct, although it is closer than the explanation in the previous exercise. 95% of future samples will be within ± 0.6 of the *true mean*, not within ± 0.6 of 26.8 (unless it happens that the true mean is 26.8). That is, future samples will not necessarily be close to the results of this sample; instead, they should be close to the *truth*.

14.39. The mistake is in saying that 95% of other polls would have results close to the results of this poll. Other surveys should be close to *the truth*—not necessarily close to the results of this survey. (Additionally, there is the suggestion that 95% means exactly "19 out of 20.")

14.40. (a) We test $H_0: \mu = 120$ min vs. $H_a: \mu > 120$ min. (b) $z = \frac{137-120}{65/\sqrt{269}} \doteq 4.29$. (c) The P-value is very small (less than 0.0001), so we have very strong evidence that students claim to study more than two hours per night.

14.41. (a) We test $H_0: \mu = 0$ vs. $H_a: \mu > 0$. (b) $z = \frac{2.35-0}{2.5/\sqrt{200}} \doteq 13.29$. (c) This value of z is far outside the range we would expect from the $N(0, 1)$ distribution.

14.42. (a) We test $H_0: \mu = 5.19$ vs. $H_a: \mu \neq 5.19$. The alternative is two-sided because we had no prior belief about the direction of the difference. (That is, before looking at the data, we had no reason to expect that the mean for hotel managers would be higher or lower than 5.19.) (b) With $\bar{y} = 5.29$, the test statistic is $z = \frac{5.29-5.19}{0.78/\sqrt{148}} \doteq 1.56$. (c) The P-value is $P = 2P(Z > 1.56) = 0.1188$. There is only weak evidence that hotel managers have different mean femininity score than the general male population. Particularly when the large sample ($n = 148$) is taken into account, we suspect that managers don't differ much from males in general (in this respect).

14.43. "$P = 0.03$" *does* mean that H_0 is unlikely, but only in the sense that the evidence (from the sample) would not occur very often if H_0 were true. P is a probability associated with the sample, not with the null hypothesis; either H_0 is true or it isn't.

14.44. If the presence of pig skulls were not an indication of wealth, then differences similar to those observed in this study would occur less than 1% of the time by chance.

14.45. $P = 0.031$ means that if cicada bodies had no effect—if nitrogen levels differed only because of natural variation among the plants—then only 3.1% of all samples would produce results similar to the ones found in this experiment.

14.46. While there was some difference in richness and total stem densities between the two areas, those differences were so small that they could easily occur by chance if the population means were identical.

14.47. In the sketch, the "significant at 1%" region includes only the dark shading ($z > 2.326$). The "significant at 5%" region of the sketch includes both the light and dark shading ($z > 1.645$).
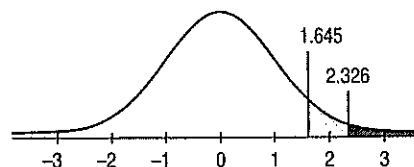
When a test is significant at the 1% level, it means that if the null hypothesis were true, outcomes similar to those seen are expected to occur less than once in 100 repetitions of the experiment or sampling. "Significant at the 5% level" means we have observed something that occurs in less than 5 out of 100 repetitions (when H_0 is true). Something that occurs "less than once in 100 repetitions" also occurs "less than 5 times in 100 repetitions," so significance at the 1% level implies significance at the 5% level (or any higher level).

The opposite statement does not hold: something that occurs "less than 5 times in 100 repetitions" is not necessarily as rare as something that occurs "less than once in 100 repetitions," so a test that is significant at 5% is not necessarily significant at 1%.

14.48. (a) The alternative hypothesis expresses the effect we expect to be true or hope to find when we plan our study. If we have no reason to expect in advance of the data that women will rate a movie more highly than men, we should use a two-sided alternative. Choosing the alternative to match the data makes it more likely that the test will find an effect. That's cheating. (b) If the one-sample z statistic is $z = 2.1$, the two-sided P-value is the probability of a result this far from zero in either direction,

$$P = 2P(Z \geq 2.1) = 2(1 - 0.9821) = 0.0358.$$

The two-sided P-value is double the one-sided value, showing again that the one-sided alternative makes it easier to find an effect.

14.49. Because a P-value is a probability, it can never be greater than 1. The correct P-value is $P(Z > 1.33) = 0.0918$.

14.50. (a) A stemplot (right) or histogram shows that the distribution is noticeably skewed to the left. The data do not appear to follow a Normal distribution. (b) STATE: What is the mean load μ required to pull apart pieces of Douglas fir?
PLAN: We will estimate μ by giving a 90% confidence interval.
SOLVE: The problem states that we are willing to take this sample to be an SRS of the population. In spite of the shape of the stemplot, we are told to suppose that this distribution is Normal with standard deviation 3000 lb. We find $\bar{x} = 30,841$ lb, so the 90% confidence interval for μ is

23	0
24	0
25	
26	5
27	
28	7
29	
30	149
31	389
32	033577
33	0126

$$\bar{x} \pm 1.645 \left(\frac{3000}{\sqrt{20}} \right) = 30,841 \pm 1103.5 = \underline{29,737 \text{ to } 31,945 \text{ lb}}.$$

CONCLUDE: We are 90% confident that the mean load required to pull apart pieces of Douglas fir is between 29,737 and 31,945 pounds.

14.51. (a) The stemplot does look reasonably Normal. **(b)** STATE: What is the mean percent change μ in spinal mineral content of nursing mothers?

PLAN: We will estimate μ by giving a 99% confidence interval.

SOLVE: The problem states that we may consider these women to be an SRS of the population. In part (a), we saw that the data appear to come from a Normal distribution. We find $\bar{x} = -3.587\%$, so the 99% confidence interval for μ is

```
−8 | 3
−7 | 80
−6 | 88552
−5 | 97633221
−4 | 9977430
−3 | 86310
−2 | 755322110
−1 | 800
−0 | 83
 0 | 234
 1 | 7
 2 | 2
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$$\bar{x} \pm 2.576 \left(\frac{2.5\%}{\sqrt{47}} \right) = -3.587\% \pm 0.939\% = -4.526\% \text{ to } -2.648\%.$$

CONCLUDE: We are 99% confident that the mean percent change in spinal mineral content is between -4.526% and -2.648%.

14.52. STATE: Is the mean load μ required to pull apart pieces of Douglas fir different from 32,000 lb? Is it different from 31,500 lb?

PLAN: To assess significance at the 10% level, we check if these loads are in a 90% confidence interval.

SOLVE: In Exercise 14.50, we found the interval to be 29,737 to 31,945 lb. **(a)** Because 32,000 lb is not in this interval, we would reject H_0: $\mu = 32,000$ lb at the 10% level (in favor of H_a: $\mu \ne 32,000$ lb). **(b)** Because 31,500 lb is in this interval, we cannot reject H_0: $\mu = 31,500$ lb at the 10% level.

CONCLUDE: The mean is significantly different (at $\alpha = 0.10$) from 32,000 lb, but not from 31,500 lb.

14.53. STATE: Do nursing mothers lose bone mineral on the average?

PLAN: Let μ be the mean percent change in spinal mineral content for the population of nursing mothers. We test H_0: $\mu = 0\%$ vs. H_a: $\mu < 0\%$; the alternative is one-sided because we suspect that nursing will reduce mineral content.

SOLVE: Assume we have a Normal distribution and an SRS. We find that $\bar{x} = -3.587\%$, so the test statistic is $z = \frac{\bar{x}-0}{2.5/\sqrt{47}} \doteq -9.84$, and the P-value is extremely small ($P = P(Z < -9.84) \doteq 0$).

CONCLUDE: This is overwhelming evidence that, on the average, nursing mothers lose bone mineral.

14.54. (a) We must assume that the 10 students can be considered to be an SRS of the population of students, and that odor thresholds are Normally distributed. A stemplot gives no reason to doubt the second condition: it is reasonably symmetric for such a small sample. **(b)** STATE: What is the mean DMS odor threshold μ among all students?

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2 | 034
2 |
3 | 01124
3 | 6
4 | 3
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PLAN: We will estimate μ by giving a 95% confidence interval.

SOLVE: We must assume that we have an SRS of the population, and that the underlying distribution is Normal with standard deviation 7 μg/l. The mean is $\bar{x} = 30.4$ μg/l, so the 95% confidence interval for μ is

$$\bar{x} \pm 1.960 \left(\frac{7}{\sqrt{10}} \right) = 30.4 \pm 4.34 = 26.06 \text{ to } 34.74 \ \mu\text{g/l}.$$

CONCLUDE: We are 95% confident that the mean DMS odor threshold among all students is between 26.06 and 34.74 μg/l.

14.55. (a) We test H_0: $\mu = 0$ vs. H_a: $\mu > 0$, where μ is the mean sensitivity difference in the population. **(b)** STATE: Does eye grease have a significant impact on eye sensitivity?
PLAN: We test the hypotheses stated in part (a).
SOLVE: The mean of the 16 differences is $\bar{x} = 0.10125$, so the test statistic is $z = \frac{0.10125 - 0}{0.22/\sqrt{16}} \doteq 1.84$. The one-sided P-value for this value of z is $P = 0.0329$.
CONCLUDE: The sample gives significant evidence (at the $\alpha = 0.05$ level) that eye grease increases sensitivity.

14.56. STATE: Is there evidence that the mean DMS threshold for untrained tasters is greater than 25 μg/l?
PLAN: We test H_0: $\mu = 25$ μg/l vs. H_a: $\mu > 25$ μg/l.
SOLVE: We find that $\bar{x} = 30.4$ μg/l, and the test statistic is $z = \frac{30.4 - 25}{7/\sqrt{10}} \doteq 2.44$, so the P-value is $P = P(Z > 2.44) = 0.0073$.
CONCLUDE: This is strong evidence against H_0; we conclude that the student's mean threshold is greater than 25 μg/l.

14.57. (a) The margin of error for 90% confidence is $1.645 \left(\frac{15}{\sqrt{72}} \right) \doteq 2.9080$, so the interval is $126.07 \pm 2.9080 = 123.16$ to 128.98. **(b)** The test statistic is $z = \frac{126.07 - 128}{15/\sqrt{72}} \doteq -1.09$, for which the two-sided P-value is $P = 0.2757$, which is greater than 0.10. **(c)** The test statistic is $z = \frac{126.07 - 129}{15/\sqrt{72}} \doteq -1.66$, for which the two-sided P-value is $P = 0.0969$, which is (barely) less than 0.10.

14.58. (a) No: 34 falls in the 95% confidence interval (28.1 to 34.9). **(b)** Yes: 35 falls (barely) outside of the 95% confidence interval.