

## Chapter 18 Solutions

- 18.1. This is a matched-pairs design: attitudes of husbands and wives may be connected, so we should keep them together.
- 18.2. This involves two independent samples: those students who read the *Wall Street Journal* ads, and those who read the *National Enquirer* ads.
- 18.3. This involves a single sample.
- 18.4. This involves two independent samples (because the result of, for example, the first measurement using the new method is independent of the first measurement using the old method).

- 18.5. (a) If the loggers had known that a study would be done, they might have (consciously or subconsciously) cut down fewer trees than they typically would, in order to reduce the impact of logging.
- (b) STATE: Does logging significantly reduce the mean number of species in a plot after 8 years?

PLAN: We test  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 > \mu_2$ , where  $\mu_1$  is the mean number of species in unlogged plots and  $\mu_2$  is the mean number of species in plots logged 8 years earlier. We use a one-sided alternative because we expect that logging reduces the number of tree species.

SOLVE: We assume that the data come from SRSs of the two populations. Stemplots (above) suggest some deviation from Normality, and a possible low outlier for the logged-plot counts. Note that these stemplots do not allow for easy comparison of the two distributions because the software used to create them used different scales (leaf units). In spite of these concerns, we proceed with the  $t$  test. The means and standard deviations are given in the Minitab output below; we compute  $SE = \sqrt{\frac{3.53^2}{12} + \frac{4.50^2}{9}} \doteq 1.813$  and  $t = \frac{17.50 - 13.67}{1.813} \doteq 2.11$ . With the conservative  $df = 8$ , we find that  $0.025 < P < 0.05$ . Note that Minitab uses the more accurate  $df = 14$  (truncated from the true computed  $df = 14.8$ ), rather than the conservative approach. For this  $df$ , we find that  $P = 0.026$ . If we remove the low outlier from the logged data, a few things change:  $\bar{x}_2 = 14.875$ ,  $s_2 \doteq 2.8504$ ,  $SE = \sqrt{\frac{3.53^2}{12} + \frac{2.85^2}{8}} \doteq 1.433$ , and  $t \doteq 1.832$ . Now we have either  $0.05 < P < 0.1$  ( $df = 7$ ) or  $P = 0.042$  ( $df = 17.2$ ).

CONCLUDE: If we use all the data, we have fairly strong evidence (significant at 5% but not at 1%) that logged plots have fewer species. If we have a reason to remove the low outlier from the logged data, the evidence is weaker, although it is still significant when we use the more accurate  $df$ .

### Minitab output

Twosample T for Species

Code	N	Mean	StDev	SE Mean
1	12	17.50	3.53	1.0
2	9	13.67	4.50	1.5

90% C.I. for  $\mu_1 - \mu_2$ : ( 0.6, 7.0)

T-Test  $\mu_1 = \mu_2$  (vs  $>$ ): T= 2.11 P=0.026 DF= 14

Unlogged	Logged
13	0   4
14	0
15	00   0
16	1   0
17	1   2
18	0   455
19	00   7
20	0   88
21	0
22	00

**18.6. STATE:** Do lean and obese people differ in the average time they spend lying down?

**PLAN:** Let  $\mu_1$  be the mean time spent lying down by the lean group, and  $\mu_2$  be the mean time for the obese group. We test  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 \neq \mu_2$ . (The question asked in the exercise suggests a two-sided alternative, but we might reasonably expect that lean people spend less time lying down, and so choose  $H_a: \mu_1 < \mu_2$ . As we shall see, that will not matter.)

**SOLVE:** See Example 18.2 for a discussion of the conditions for inference. The back-to-back stemplot on the right shows some irregularity, but no clear departures from Normality. They also suggest little difference between the two groups—an impression which is supported by the means:

Lean		Obese
9	3	
	4	1
	4	
5	4	44
6	4	6
8	4	
10	5	001
33	5	23
5	5	
6	5	6

	$n$	$\bar{x}$	$s$
Lean	10	501.6461	52.0449
Obese	10	491.7426	46.5932

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \doteq 22.0898$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} \doteq 0.448$$

The standard error and the test statistic are given above. With such a small  $t$  value, the  $P$ -value will obviously be quite large regardless of the degrees of freedom; in fact,  $P = 0.6596$  ( $df = 17.8$ ) or  $P = 0.6647$  ( $df = 9$ ).

**CONCLUDE:** There is no reason to reject the hypothesis that lean and moderately obese people spend (on the average) the same amount of time lying down.

**18.7.** Use the means and the standard error  $SE \doteq 1.813$  found in the solution to Exercise 18.5, and critical value  $t^* = 1.860$  ( $df = 8$ ) or  $t^* = 1.755$  ( $df = 14.8$ ). The 90% confidence interval is  $\bar{x}_1 - \bar{x}_2 \pm t^*SE$ , which gives 0.46 to 7.20 species ( $df = 8$ ) or 0.65 to 7.02 species ( $df = 14.8$ ). Minitab's result, shown in the solution to Exercise 18.5 above, is based on the truncated  $df = 14$ .

**18.8.** To test  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 > \mu_2$  (which is equivalent to the hypotheses stated in Figure 18.5), we have  $t \doteq 0.9889$ ,  $df \doteq 4.65$ , and  $P = 0.1857$ . This gives us little reason to doubt that fabric buried two weeks and fabric buried 16 weeks have the same mean breaking strength.

**18.9. (a)** Back-to-back stemplots of the time data are shown below on the left. They appear to be reasonably Normal, and the discussion in the exercise justifies our treating the data as independent SRSs, so we can use the  $t$  procedures.

We wish to test  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 < \mu_2$ , where  $\mu_1$  is the population mean time in the restaurant with no scent, and  $\mu_2$  is the mean time with a lavender odor. The means and standard deviations (in minutes), as well as the standard error and test statistic, are

	$n$	$\bar{x}$	$s$
No scent	30	91.26	14.9296
Lavender	30	105.7	13.1048

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \doteq 3.6269$$

$$t = \frac{91.26 - 105.7}{3.6269} \doteq -3.98$$

For this value of  $t$ ,  $P < 0.0005$  ( $df = 29$ ) or  $P = 0.0001$  ( $df = 57.041$ ). This is strong evidence that customers stay longer when the lavender odor is present. **(b)** Back-to-back

stemplots of the spending data are below on the right. The distributions are skewed and have many gaps. Again we test  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 < \mu_2$ , where  $\mu_1$  is the population mean spending with no scent, and  $\mu_2$  is the mean spending with a lavender odor. The means and standard deviations (in euros), as well as the standard error and test statistic, are

	$n$	$\bar{x}$	$s$
No scent	30	17.5133	2.3588
Lavender	30	21.1233	2.3450

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \doteq 0.6073$$

$$t = \frac{17.5133 - 21.1233}{0.6073} \doteq -5.95$$

For this value of  $t$ ,  $P < 0.0005$  ( $df = 29$ ) or  $P$  is very small ( $df = 57.998$ ). This is strong evidence that customers spend more when the lavender odor is present.

No scent		Lavender	No scent		Lavender
98	6		9	12	
322	7			13	
965	7	6		14	
44	8		999999999999999	15	
7765	8	89		16	
32221	9	234		17	
86	9	578	555555555555	18	5555555555
31	10	1234		19	
9776	10	5566788999	5	20	7
	11	4	9	21	5599999999
85	11	6		22	3558
1	12	14		23	
	12	69		24	99
	13		5	25	59
	13	7			

18.10. (a) The “compressed” stemplot shows no particular cause for concern. The “intermediate” stemplot shows the skewness and outlier described in the text. Note that these stemplots do not allow for easy comparison of the two distributions because the software used to create them used different scales (leaf units). (b) We wish to test  $H_0: \mu_C = \mu_I$  vs.  $H_a: \mu_C < \mu_I$ . The summary statistics are:

	$n$	$\bar{x}$	$s$
Compressed	20	2.9075	0.1390
Intermediate	19	3.2874	0.2397

Compressed	Intermediate
26   8	2   99
27	3   0111111
27   6888	3   2333
28   122	3   4445
28   66	3   6
29   024	3   8
29   68	4
30   0	4   2
30   88	
31	
31   68	

We compute  $SE = \sqrt{\frac{0.1390^2}{20} + \frac{0.2397^2}{19}} \doteq 0.06317$  and  $t = \frac{2.9075 - 3.2874}{0.06317} \doteq -6.013$ . Regardless of the chosen  $df$  (conservative 18, or the software value 28.6), the  $P$ -value is very small; this is very significant evidence that the mean penetrability for compressed soil is lower than the mean for intermediate soil.

**Minitab output**

```

Twosample T for Comp vs Inter
      N      Mean    StDev   SE Mean
Comp  20      2.908    0.139    0.031
Inter 19      3.287    0.240    0.055

95% C.I. for mu Comp - mu Inter: ( -0.509, -0.250)
T-Test mu Comp = mu Inter (vs <): T= -6.01 P=0.0000 DF= 28
    
```

18.11. We have two small samples (each  $n = 4$ ), so  $t$  procedures are not reliable unless both distributions are Normal.

18.12. Use the means and the standard error  $SE \doteq 0.06317$  found in the solution to Exercise 18.10, and critical value  $t^* = 2.101$  ( $df = 18$ ) or  $t^* = 2.047$  ( $df = 28.6$ ). The 95% confidence interval is  $\bar{x}_1 - \bar{x}_2 \pm t^*SE$ , which gives  $-0.5126$  to  $-0.2471$  ( $df = 18$ ) or  $-0.5092$  to  $-0.2506$  ( $df = 28.6$ ). Minitab's result, shown in the solution to Exercise 18.10, is based on the truncated  $df = 28$ .

18.13. Here are the details of the computations:

$$SE_F = \frac{12.6961}{\sqrt{31}} \doteq 2.2803$$

$$SE_M = \frac{12.2649}{\sqrt{47}} \doteq 1.7890$$

$$SE = \sqrt{SE_F^2 + SE_M^2} \doteq 2.8983$$

$$df = \frac{SE^4}{\frac{1}{30} \left( \frac{12.6961^2}{31} \right)^2 + \frac{1}{46} \left( \frac{12.2649^2}{47} \right)^2} = \frac{70.565}{1.1239} \doteq 62.8$$

$$t = \frac{55.5161 - 57.9149}{SE} \doteq -0.8276$$

18.14. The means and standard deviations are not given in Figure 18.5, but they are easily computed. They are shown in the table on the right, along with  $s_1^2/n_1$  and  $s_2^2/n_2$ , which we need to find the standard error and degrees of freedom. Here are the details of the computations:

Group	$\bar{x}$	$s$	$s^2/n$
2 weeks	123.8	4.6043	4.24
18 week	116.4	16.0873	51.76

$$SE = \sqrt{4.24 + 51.76} \doteq 7.4833$$

$$df = \frac{SE^4}{\frac{1}{4}(4.24)^2 + \frac{1}{4}(51.76)^2} = \frac{3136}{674.2688} \doteq 4.651$$

$$t = \frac{123.8 - 116.4}{SE} \doteq 0.9889$$

18.15. Reading from the software output shown in Exercise 18.13, we find that there was no significant difference in mean Self-Concept Scale scores for men and women ( $t = -0.8276$ ,  $df = 62.8$ , and  $P = 0.4110$ ).

18.16. (a) We have a single sample, and only one score from each member of the sample.

18.17. (c) We consider the boys and girls as being two independent samples.

18.18. (b) Measure the amount of pollutant in each carp using both methods, and examine the differences between those pairs of measurements.

18.19. (b) This is the definition of robust.

18.20. (a) We have two independent samples, each of size  $n = 8$ , so  $df = 7$ .

18.21. (b)  $SE = \sqrt{\frac{9^2}{8} + \frac{13^2}{8}} \doteq 5.59$ , and  $t = \frac{105-89}{SE} \doteq 2.86$ .

18.22. (c) Random-digit dialing should produce something close to an SRS, and the samples are easily large enough to overcome non-Normality (especially because the scale 0 to 20 limits skewness and outliers in the distribution of scores).

18.23. (a) Our alternative hypothesis should express our suspicion that men are more prone to road rage than women. (We assume that a high road-rage score means a subject is *more* prone to road rage.)

18.24. (b) Using Table C, refer to  $df = 100$ , where we see that  $3.174 < t < 3.390$ . This means that  $P$  is between 0.0005 and 0.001.

18.25. (a) To test the belief that women talk more than men, we use a one-sided alternative:  $H_0: \mu_M = \mu_F$  vs.  $H_a: \mu_M < \mu_F$ . (b)–(d) The  $t$  statistics, degrees of freedom, and  $P$ -values are given in the table below. The two-sample  $t$  statistics are  $\frac{\bar{x}_F - \bar{x}_M}{\sqrt{s_F^2/n_F + s_M^2/n_M}}$ , so positive values would tend to support  $H_a$ . The conservative  $df$  is the smaller sample size, minus 1.

Study	$t$	df	Table C values	$P$ -value
1	-0.248	55	$ t  < 0.679$	$P > 0.25$
2	1.507	19	$1.328 < t < 1.729$	$0.05 < P < 0.10$

For Study 1, we use  $df = 50$  in Table C. (e) The first study gives no support to the belief that women talk more than men; the second study gives only moderate (not significant) support.

18.26. (a) The two standard deviations are  $s_u = 7\sqrt{9} = 21$  and  $s_r = 10\sqrt{11} \doteq 33.1662$ .

(b) Using Option 2, with sample sizes 9 and 11, we take  $df = 8$ . (c) We find  $SE = \sqrt{7^2 + 10^2} \doteq 12.2065$  g. With  $t^* = 1.860$  ( $df = 8$ ), the 90% confidence interval is  $(59 - 32) \pm t^*SE \doteq 4.301$  to 49.699 g. (For comparison, using the software  $df \doteq 17.08$ ,  $t^* = 1.7392$  and the interval is 5.771 to 48.229 g.)

18.27. (a) If  $SEM = s/\sqrt{n}$ , then  $s = SEM \times \sqrt{n}$ . Arithmetic gives the table on the right. (b) The smaller sample has  $n = 6$ , so the conservative  $df$  is 5. (c)  $SE = \sqrt{1.56^2 + 2.68^2} \doteq 3.1010$ , so  $t = \frac{26.9-11.9}{SE} \doteq 4.837$ . (d) Because  $4.773 < t < 5.893$ , the two-sided  $P$ -value is between 0.002 and 0.005. (Software gives  $P = 0.0047$ .)

Location	$n$	$\bar{x}$	$s$
Oregon	6	26.9	3.82
California	7	11.9	7.09

18.28. (a) Parents who choose a Montessori school probably have different attitudes about education than other parents. (b) Over 72% of Montessori parents participated in the study, compared to less than 28% of the other parents. (c) To test  $H_0: \mu_M = \mu_C$  vs.

$H_a: \mu_M \neq \mu_C$ , we find  $SE = \sqrt{\frac{3.11^2}{19} + \frac{4.19^2}{17}} \doteq 1.0122$ , so the two-sample test statistic is  $t = \frac{19-17}{SE} \doteq 1.976$ . This is not quite significant the 5% level:  $P = 0.0545$  with  $df = 43.5$ , or  $0.05 < P < 0.10$  with  $df = 24$ .

18.29. (a) A placebo is an inert pill that allows researchers to account for any psychological benefit (or detriment) the subject might get from taking a pill. (b) Neither the subjects nor the researchers who worked with them knew who was getting ginkgo extract; this prevents expectations or prejudices from affecting the evaluation of the effectiveness of the treatment. (c)  $SE = \sqrt{\frac{0.01462^2}{21} + \frac{0.01549^2}{18}} \doteq 0.0048$ , so the two-sample test statistic is  $t = \frac{0.06383 - 0.05342}{SE} \doteq 2.147$ . This is significant at the 5% level:  $P = 0.0387$  ( $df \doteq 35.35$ ) or  $0.04 < P < 0.05$  ( $df = 17$ ). Those who took ginkgo extract had significantly more misses per line.

18.30. "Do Hispanic and Anglo bank customers differ?" calls for two-sided tests:  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 \neq \mu_2$  (for both reliability and empathy). The table below gives the standard errors,  $t$  statistics, degrees of freedom, and  $P$ -values for both tests. (The conservative  $df$  would be 85, but for use with Table C, we must take  $df = 80$ .) Both results are very significant; there is strong evidence that Anglos value reliability more than Hispanics do, and that Hispanics value empathy more than Anglos do.

	$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}$	Conservative		Software	
			df	$P$	df	$P$
Reliability	0.1182	3.892	80	< 0.001	143.69	0.00015
Empathy	0.1196	-3.595	80	< 0.001	171.08	0.00042

18.31. To test  $H_0: \mu_A = \mu_E$  vs.  $H_a: \mu_A \neq \mu_E$ , we find  $SE = \sqrt{\frac{0.60^2}{12} + \frac{0.57^2}{9}} \doteq 0.2571$ , so the two-sample test statistic is  $t = \frac{1.92 - 1.74}{SE} \doteq 0.7$ . This is clearly not significant:  $P > 0.5$  ( $df = 8$ ) or  $P = 0.4930$  ( $df = 17.85$ ). There is no evidence of a difference in mean stress level between Asian and European mothers.

18.32. (a) The appropriate test is the matched-pairs test because a student's score on Try 1 is certainly correlated with his/her score on Try 2. (b) To test  $H_0: \mu = 0$  vs.  $H_a: \mu > 0$ , we compute  $t = \frac{29 - 0}{59/\sqrt{427}} \doteq 10.16$  with  $df = 426$ , which is certainly significant ( $P < 0.0005$ ). Coached students do improve their scores. The TI-83 output screen (on the right) shows that the  $P$ -value is, in fact, *much* smaller than 0.0005! (c) Table C gives  $t^* = 2.626$  for  $df = 100$ , while software gives  $t^* = 2.587$  for  $df = 426$ . The confidence interval is  $29 \pm t^*59/\sqrt{427} = 21.50$  to  $36.50$  points (or 21.61 to 36.39, using the software critical value). The TI-83 output confirms the more exact interval.

```
T-Test
μ>0
t=10.1568707
p=3.773541E-22
x̄=29
Sx=59
n=427
```

```
TInterval
(21.612, 36.388)
x̄=29
Sx=59
n=427
```

- 18.33. (a) The hypotheses are  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 > \mu_2$ , where  $\mu_1$  is the mean gain among all coached students, and  $\mu_2$  the mean gain among uncoached students. We find  $SE = \sqrt{59^2/427 + 52^2/2733} \doteq 3.0235$  and  $t = \frac{29-21}{3.0235} \doteq 2.646$  with conservative  $df = 426$  or software  $df = 534.45$ . Comparing with  $df = 100$  critical values in Table C, we find  $0.0025 < P < 0.005$ ; software gives  $P \doteq 0.004$  for  $df = 534.45$ . There is evidence that coached students had a greater average increase. (b) The 99% confidence interval is  $8 \pm 3.0235t^*$ , where  $t^*$  equals 2.626 ( $df = 100$ , from Table C) or 2.585 ( $df = 534.45$ ). This gives either 0.06 to 15.94 points, or 0.184 to 15.816 points. (c) Increasing one's score by 0 to 16 points is not likely to make a difference in being granted admission to, or receiving scholarships from, any colleges.

```
2-SampTTest
μ1>μ2
t=2.645931232
P=.0041933105
df=534.4495612
x̄1=29
x̄2=21
```

```
2-SampTInt
(.18405, 15.816)
df=534.4495612
x̄1=29
x̄2=21
s*1=59
s*2=52
```

- 18.34. This was an observational study, not an experiment. The students (or their parents) chose whether or not to be coached; students who choose coaching might have other motivating factors that help them do better the second time. For example, perhaps students who choose coaching have some personality trait that also compels them to try harder the second time.

- 18.35. (a) The back-to-back stemplots on the right confirm the description given in the text. (b) Means and standard deviations are given in the table on the right. Testing  $H_0: \mu_G = \mu_B$  vs.  $H_a: \mu_G \neq \mu_B$ , we find  $SE \doteq 3.1138$  and  $t \doteq 1.64$ . This gives  $0.10 < P < 0.20$  ( $df = 30$ ) or  $P = 0.1057$  ( $df = 56.9$ ), so it is not strong enough evidence to reject  $H_0$ .

	$n$	$\bar{x}$	$s$
Girls	31	105.84	14.271
Boys	47	110.96	12.121

  

Girls		Boys
42	7	
	7	79
	8	
	8	
96	9	03
31	9	77
86	10	0234
433320	10	556667779
875	11	00001123334
44422211	11	556899
98	11	03344
0	12	67788
8	12	
20	13	
	13	6

18.36. (a) Based on the stemplots, the  $t$  procedures should be safe. Both the stemplots and the means suggest that customers stayed (very slightly) longer when there was no odor. (b) Testing  $H_0: \mu_N = \mu_L$  vs.  $H_a: \mu_N \neq \mu_L$ , we find  $SE \doteq 3.9927$  and  $t \doteq 0.371$ . This is not at all significant:  $P > 0.5$  ( $df = 27$ ) or  $P = 0.7121$  ( $df = 55.4$ ). We cannot conclude that mean time in the restaurant is different when the lemon odor is present.

	$n$	$\bar{x}$	$s$
No odor	30	91.2667	14.9296
Lemon	28	89.7857	15.4377

No odor		Lemon
	5	6
	6	03
98	6	
322	7	34
965	7	58
44	8	33
7765	8	8889
32221	9	0144
86	9	677
31	10	14
9776	10	5688
	11	23
85	11	
1	12	

18.37. The 95% confidence interval is  $(110.96 - 105.84) \pm t^*SE$ , where  $SE \doteq 3.1138$  and  $t^* = 2.042$  ( $df = 30$ ) or  $2.003$  ( $df = 56.9$ ). These intervals are  $-1.24$  to  $11.48$  or  $-1.12$  to  $11.35$ .

18.38. (a) The sample means in the table on the right suggest that the Permafresh swatches are slightly stronger. (The second Permafresh line was computed with the low outlier omitted.) (b) Shown on the right are back-to-back stemplots for the two processes. (c) With the means and standard deviations listed in the table, we find the following test statistics and  $P$ -values:

	$n$	$\bar{x}$	$s$
Permafresh	5	29.54	1.1675
Permafresh*	4	30.025	0.4992
Hylite	5	25.20	2.6693

Permafresh		Hylite
	22	1
	23	9
	24	2
	25	
	26	
6	27	0
	28	8
95	29	
70	30	

	$t$	df	Conservative $P$	Software df	Software $P$
All points	3.33	4	$0.02 < P < 0.04$	5.48	0.0184
Outlier removed	3.96	3	$0.02 < P < 0.04$	4.35	0.0147

The mild outlier in the Permafresh sample had almost no effect on our conclusion. Despite the small samples, there is good evidence (significant at  $\alpha = 0.05$ ) that the mean breaking strengths of the two processes differ.

18.39. (a) The Hylite mean is greater than the Permafresh mean. (b) Shown on the right are back-to-back stemplots for the two processes, which confirm that there are no extreme outliers. (c) We find  $SE \doteq 1.334$  and  $t \doteq -6.296$ , for which the  $P$ -value is  $0.002 < P < 0.005$  ( $df = 4$ ) or  $0.0003$  ( $df = 7.779$ ). There is very strong evidence of a difference between the population means. As we might expect, the stronger process (Permafresh) is less resistant to wrinkles.

	$n$	$\bar{x}$	$s$
Permafresh	5	134.8	1.9235
Hylite	5	143.2	2.2804

Permafresh		Hylite
	2	
	13	
54	13	
76	13	
	13	
	14	11
	14	3
	14	5
	14	6

18.40. The 90% confidence interval is  $\bar{x}_1 - \bar{x}_2 \pm t^*SE$ , where  $t^* = 2.132$  ( $df = 4$ ) or  $t^* = 1.981$  ( $df = 5.476$ ). This gives either

$$4.34 \pm 2.778 = 1.562 \text{ to } 7.118 \text{ pounds (df = 4), or}$$

$$4.34 \pm 2.581 = 1.759 \text{ to } 6.921 \text{ pounds (df = 5.476).}$$

As usual, the conservative interval is wider than the more accurate software result.

18.41. The 90% confidence interval is  $\bar{x}_1 - \bar{x}_2 \pm t^*SE$ , where  $t^* = 2.132$  ( $df = 4$ ) or  $t^* = 1.867$  ( $df = 7.779$ ). This gives either

$$-8.4 \pm 2.844 = -11.244^\circ \text{ to } -5.556^\circ \text{ (df = 4), or}$$

$$-8.4 \pm 2.491 = -10.891^\circ \text{ to } -5.909^\circ \text{ (df = 7.779).}$$

As usual, the conservative interval is wider than the more accurate software result.

18.42. This is a two-sample  $t$  statistic, comparing two independent groups (supplemented and control). Using the conservative  $df = 5$ ,  $t = -1.05$  would have a  $P$ -value between 0.30 and 0.40, which (as the report said) is not significant. ( $t = -1.05$  would not be significant for any  $df$ .)

18.43. To test  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 \neq \mu_2$ , we

$$\text{find } SE = \sqrt{\frac{3.10934^2}{6} + \frac{3.92556^2}{7}} \doteq 1.95263, \text{ and}$$

$$t = \frac{4.0 - 11.3}{SE} \doteq -3.74. \text{ The two-sided } P\text{-value is either}$$

$0.01 < P < 0.02$  ( $df = 5$ ) or  $0.0033$  ( $df = 10.96$ ), agreeing with the stated conclusion (a significant difference).

	$n$	$\bar{x}$	$s$
Control	6	4.0	3.10934
Supplemented	7	11.3	3.92556

18.44. These are paired  $t$  statistics: for each bird, the number of days behind the caterpillar peak was observed, and the  $t$  values were computed based on the pairwise differences between the first and second years.

For the control group,  $df = 5$ , and for the supplemented group,  $df = 6$ . The control  $t$  is not significant (so the birds in that group did *not* "advance their laying date in the second year"), while the supplemented group  $t$  is significant with one-sided  $P = 0.0195$  (so those birds did change their laying date).

18.45. PLAN: We test  $H_0: \mu_T = \mu_C$  vs.  $H_a: \mu_T > \mu_C$ , using a one-sided alternative because we suspect that the treatment group will wait longer before asking for help.

SOLVE: We must assume that the data comes from an SRS of the intended population (whatever we consider that to be); we cannot check this with the data. The back-to-back stemplot (right) shows some irregularity in the treatment times, and skewness in the control times. We hope that our equal and moderately large sample sizes will overcome any deviation from Normality.

With the means and standard deviations listed in the table, we find  $SE \doteq 50.7602$  and  $t \doteq 2.521$ , for which  $0.01 < P < 0.02$  ( $df = 16$ ) or  $P = 0.0088$  ( $df = 28.27$ ).

CONCLUDE: There is strong evidence that the treatment group waited longer to ask for help on the average.

	$n$	$\bar{x}$	$s$
Treatment	17	314.0588	172.7898
Control	17	186.1176	118.0926

Treatment		Control
65	0	5689
3	1	012444
976	1	58
44	2	
5	2	79
	3	
6	3	7
44	4	01
876	4	
3	5	
	5	
0	6	

18.46. PLAN: We test  $H_0: \mu_A = \mu_P$  vs.  $H_a: \mu_A > \mu_P$ ; the one-sided alternative is suggested by the statement of the exercise—and presumably by a suspicion that active learning is better.

SOLVE: We must assume that the data comes from an SRS of the population of learning-impaired children; we cannot check this with the data. We also assume that the data are close to Normal. The back-to-back stemplot (right) shows a high outlier and some skewness in the “active” scores, but with reasonably large (and equal) sample sizes such as those we have here, we can allow some variation from Normality. With the means and standard deviations listed in the table, we find  $SE \doteq 1.5278$  and  $t \doteq 4.28$ . With either  $df = 23$  or  $df = 39.1$ ,  $P < 0.0005$ .

CONCLUDE: There is very strong evidence that active learning results in more correct identifications.

	$n$	$\bar{x}$	$s$
Active	24	24.4167	6.31022
Passive	24	17.8750	4.02506

Active		Passive
	1	223
5	1	45555
76	1	66777
	1	889
111100	2	00111
332	2	
4444	2	5
7	2	66
9888	2	
1	3	
	3	
5	3	
	3	
	4	
	4	
4	4	

18.47. PLAN: Compare mean gains by testing  $H_0: \mu_1 = \mu_2$  vs.  $H_a: \mu_1 > \mu_2$ , and by finding a 90% confidence interval for  $\mu_1 - \mu_2$ .

	$n$	$\bar{x}$	$s$
Treatment	10	11.4	3.1693
Control	8	8.25	3.6936

SOLVE: We assume that we have two SRSs, and that the distributions of score improvements are Normal. Shown on the right are stemplots of the differences ("after" minus "before") for the two groups; the samples are too small to assess Normality, but we can see that there are no outliers. With the means and standard deviations listed in the table, we find  $SE \doteq 1.646$  and  $t \doteq 1.914$ ,

Treatment		Control
	0	455
76	0	7
	0	8
110	1	1
332	1	2
5	1	4
6	1	

for which the  $P$ -value is  $0.025 < P < 0.05$  ( $df = 7$ ) or  $0.0382$  ( $df = 13.92$ ). The 90% confidence interval is  $(11.40 - 8.25) \pm t^*SE$ , where  $t^* = 1.895$  ( $df = 7$ ) or  $t^* = 1.762$  ( $df = 13.92$ ): either 0.03 to 6.27 points, or 0.25 to 6.05 points.

CONCLUDE: We have evidence (significant at 5%) that the encouraging subliminal message led to a greater improvement in math scores. We are 90% confident that this increase is between 0.03 and 6.27 points (or 0.25 and 6.05 points).

18.48. (a) The 90% confidence interval is  $(24.4167 - 17.8750) \pm t^*SE$ , where  $SE \doteq 1.5278$  and  $t^* = 1.714$  ( $df = 23$ ) or  $1.685$  ( $df = 39.05$ ). These intervals are 3.92 to 9.16 Blissymbols, or 3.97 to 9.12 Blissymbols. (b) With  $t^* = 1.714$  from a  $t(23)$  distribution, the 90% confidence interval is  $24.4167 \pm (1.714)(6.31022/\sqrt{24}) = 22.2$  to 26.6 Blissymbols. (Note that this is a one-sample question.)

18.49. PLAN: Compare mean length by testing  $H_0: \mu_r = \mu_y$  vs.  $H_a: \mu_r \neq \mu_y$ , and by finding a 95% confidence interval for  $\mu_r - \mu_y$ .

	$n$	$\bar{x}$	$s$
Red	23	39.7113	1.7988
Yellow	15	36.1800	0.9753

SOLVE: We must assume that the data comes from an SRS. We also assume that the data are close to Normal. The back-to-back stemplots (right) show some skewness in the red lengths, but the  $t$  procedures should be reasonably safe. With the means and standard deviations listed in the table, we find  $SE \doteq 0.4518$  and  $t \doteq 7.817$ . With either  $df = 14$  or  $df = 35.1$ ,  $P < 0.001$ . The 95% confidence interval is  $(39.711 - 36.180) \pm t^*SE$ , where  $t^* = 2.145$  ( $df = 14$ ) or  $t^* = 2.030$  ( $df = 35.1$ ): either 2.562 to 4.500 mm, or 2.614 to 4.448 mm.

Red		Yellow
	34	56
	35	146
	36	0015678
9874	37	01
8722100	38	1
761	39	
65	40	
9964	41	
10	42	
0	43	

CONCLUDE: We have very strong evidence that the two varieties differ in mean length. We are 95% confident that the mean red length minus yellow length is between 2.562 and 4.500 mm (or 2.614 and 4.448 mm).

18.50. Because this exercise asks for a "complete analysis," without suggesting hypotheses or confidence levels, student responses may vary. This solution gives 95% confidence intervals for the means in parts (a) and (b), and performs a hypothesis test and gives a 95% confidence interval for part (c). Note that the first two problems call for single-sample  $t$  procedures (Chapter 17), while the last uses the Chapter 18 procedures. Student answers should be formatted according to the "four-step process" of the text; these answers are not, but can be used to check student results.

