Chapter 19 Solutions

19.1. **(a)** The population is "all college students" (or something similar). p is the proportion of the population who say they pray at least once in a while. **(b)** $\hat{p} = \frac{107}{127} \doteq 0.8425$.

19.2. The proportion who would say "yes" is approximately Normal with mean $p = 0.4$ and standard deviation $\sqrt{\frac{p(1-p)}{1500}} \doteq 0.01265$, so 99.7% of sample proportions will be within 3 standard deviations of the mean: $.4 \pm 0.038 = 0.362$ to 0.438.

19.3. **(a)** The sample proportion $\hat{p}$ is approximately Normal with mean $p = 0.75$ and standard deviation $\sqrt{\frac{p(1-p)}{1000}} \doteq 0.01369$. **(b)** With $n = 4000$, the standard deviation drops to $\sqrt{\frac{p(1-p)}{4000}} \doteq 0.00685$. (Quadrupling the sample size cuts the standard deviation in half.)

19.4. There were only 5 or 6 "successes" in the sample (because 5/2673 and 6/2673 both round to 0.2%).

19.5. **(a)** The survey excludes residents of the northern territories, as well as those who have no phones or have only cell phone service. **(b)** $\hat{p} = \frac{1288}{1505} \doteq 0.8558$, so $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/1505} \doteq 0.009055$, and the 95% confidence interval is $0.8558 \pm (1.96)(0.009055) = 0.8381$ to 0.8736.

19.6. State: What proportion p of students who retake the SAT paid for coaching?

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1-PropZInt
 (.11946,.15079)
 p̂=.1351265823
 n=3160
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Plan: We will find a 99% confidence interval for p.
Solve: We are told this is a random sample; we assume this means it is at least close to an SRS. Both the number of successes (427) and the number of failures (2733) are much greater than 15. We compute $\hat{p} = \frac{427}{3160} \doteq 0.1351$ and $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/3160} \doteq 0.006081$, so the 99% confidence interval is $0.1351 \pm (2.576)(0.006081) = 0.1195$ to 0.1508. This agrees with the TI-83 output on the right.
Conclude: We are 99% confident that the proportion of coaching among students who retake the SAT is between 0.1195 and 0.1508.

19.7. **(a)** The large-sample interval should not be used because the number of "successes" is only 9. **(b)** The sample size is 102, with 11 successes. The plus four estimate is $\tilde{p} = \frac{9+2}{98+4} \doteq 0.1078$. **(c)** The 90% confidence interval is

$$\tilde{p} \pm 1.645\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}} = 0.1078 \pm 0.0505 = 0.0573 \text{ to } 0.1584.$$

19.8. (a) With the sample proportion $\hat{p} = \frac{385}{487} \doteq 0.7906$, the 95% confidence interval is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.7906 \pm 0.03614 = 75.4\% \text{ to } 82.7\%.$$

(b) The plus four estimate is $\tilde{p} = \frac{385+2}{487+4} \doteq 0.7882$, and the plus four interval is

$$\tilde{p} \pm 1.96\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}} = 0.7882 \pm 0.03614 = 75.2\% \text{ to } 82.4\%.$$

As the exercise states, the plus four interval is pulled away from 100%, by 0.2% on the low end and 0.3% on the high end. (If we instead round to the nearest 0.01%, both the upper and lower limits decrease by 0.24%.)

19.9. (a) The sample proportion is $\hat{p} = \frac{20}{20} = 1$, so $\text{SE}_{\hat{p}} = 0$. The margin of error would therefore be 0 (regardless of the confidence level), so large-sample methods give the interval 1 to 1. (b) The plus four estimate is $\tilde{p} = \frac{20+2}{20+4} \doteq 0.9167$, and the plus four interval is

$$\tilde{p} \pm 1.96\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}} = 0.9167 \pm 0.11058 = 0.8061 \text{ to } 1.0272.$$

Because proportions can be no larger than 1, use 1 for the upper limit.

19.10. (a) We find $\hat{p} = \frac{221}{270} \doteq 0.8185$ and $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/270} \doteq 0.02346$, so the margin of error for 95% confidence is $(1.96)(0.02346) \doteq 0.0460$. (b) For a ± 0.03 margin of error, we need $n = \left(\frac{1.96}{0.03}\right)^2 p^*(1-p^*) = 4268.\bar{4} \cdot p^*(1-p^*)$. There are several ways we could take p^* from the pilot study: we could simply take $p^* = \hat{p}$, or we could try values of p^* that are slightly smaller than $\hat{p}$, taking into account the margin of error found in (a). The table on the right summarizes the sample size for various choices of p^*.

p^*	n
$\hat{p}$	634.06
0.80	682.95
0.75	800.33

19.11. $n = \left(\frac{1.645}{0.04}\right)^2 (0.75)(0.25) \doteq 317.1$—use $n = 318$.

19.12. State: Does $\hat{p} = \frac{83}{200} = 0.415$ give significant evidence that the probability of heads from spinning a coin is different from 0.5?

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1-PropZTest
 prop≠.5
 z=-2.404163056
 p=.0162095291
 p̂=.415
 n=200
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Plan: Let p be the proportion of heads from a spun coin. We test H_0: $p = 0.5$ vs. H_a: $p \neq 0.5$; the alternative is two-sided because (prior to looking at the data) we had no suspicion that coin spinning would favor heads or tails.

Solve: We view the 200 spins as an SRS. The expected counts np_0 and $n(1-p_0)$ are both 100—large enough to proceed. With $\hat{p} = 0.415$, the test statistic is $z = (0.415 - 0.5)/\sqrt{\frac{(0.5)(0.5)}{200}} \doteq -2.40$. The P-value is 0.0164 (from Table A) or 0.0162 (software).

Conclude: We have pretty strong evidence—significant at $\alpha = 0.05$, though not at $\alpha = 0.01$—against equal probabilities.

19.13. State: Does $\hat{p} = \frac{22}{32} = 0.6875$ give significant evidence that the "best face" wins more than half the time?

Plan: Let p be the proportion of races won by the candidate with the better face. We test

H_0: $p = 0.5$ vs. H_a: $p > 0.5$; the alternative is one-sided because we suspect (even before seeing the data) that the better-face candidate has an advantage.
SOLVE: We view the 32 races as an SRS. The expected counts np_0 and $n(1 - p_0)$ are both 16—large enough to proceed. With $\hat{p} = 0.6875$, the test statistic is $z = (0.6875 - 0.5)/\sqrt{\frac{(0.5)(0.5)}{32}} \doteq 2.12$. The P-value is 0.0170 (from Table A) or 0.0169 (software).
CONCLUDE: We have pretty strong evidence—significant at $\alpha = 0.05$, though not at $\alpha = 0.01$—that the candidate with the better face wins more than half the time.

19.14. **(a)** The sample size (10) is too small; it gives $np_0 = n(1 - p_0) = 5$. **(b)** The expected number of "no" responses is too small: $n(1 - p_0) = 200(0.01) = 2$.

19.15. (b) The mean of the distribution of $\hat{p}$ is the population proportion p. (That is, $\hat{p}$ is an unbiased estimate of p.)

19.16. (c) The standard deviation is $\sqrt{\frac{(0.3)(0.7)}{757}} \doteq 0.01666$.

19.17. (c) $\hat{p} = \frac{273}{757} \doteq 0.36$.

19.18. (b) The margin of error for 95% confidence is $1.96 \cdot \text{SE} = 1.96\sqrt{\frac{(0.36)(0.64)}{757}} \doteq 0.034$.

19.19. (c) $n = \left(\frac{1.96}{0.02}\right)^2 (0.5)(0.5) = 2401$.

19.20. (a) $\tilde{p} = \frac{55}{104} \doteq 0.529$ and the margin of error is $1.96\sqrt{\frac{\tilde{p}(1-\tilde{p})}{104}} \doteq 0.096$.

19.21. (a) The reported margin of error accounts only for random variation in the responses.

19.22. (a) The alternative hypothesis expresses the idea "more than half think their job prospects are good."

19.23. (c) $z = (0.53 - 0.5)/\sqrt{\frac{(0.5)(0.5)}{100}} = 0.6$.

19.24. (a) The margin of error in (almost all) publicly reported polls is with 95% confidence.

19.25. **(a)** The survey excludes those who have no phones or have only cell phone service. **(b)** With the sample proportion $\hat{p} = \frac{848}{1010} \doteq 0.8396$, the large sample 95% confidence interval is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.8396 \pm 0.02263 = 0.8170 \text{ to } 0.8622.$$

If we instead use the plus four method, $\tilde{p} = \frac{848+2}{1010+4} \doteq 0.8383$, $\text{SE}_{\tilde{p}} \doteq 0.01156$, the margin of error is $1.960\text{SE}_{\tilde{p}} \doteq 0.02266$, and the 95% confidence interval is 0.8156 to 0.8609.

19.26. We estimate that $\hat{p} = \frac{19}{172} \doteq 0.1105$, $\text{SE}_{\hat{p}} \doteq 0.02390$, the margin of error is $1.960\text{SE}_{\hat{p}} \doteq 0.04685$, and the 95% confidence interval is 0.0636 to 0.1573.
If we instead use the plus four method, $\tilde{p} = \frac{19+2}{172+4} \doteq 0.1193$, $\text{SE}_{\tilde{p}} \doteq 0.02443$, the margin of error is $1.960\text{SE}_{\tilde{p}} \doteq 0.04789$, and the 95% confidence interval is 0.0714 to 0.1672.

19.27. **(a)** With $\hat{p} = \frac{848}{1010} \doteq 0.8396$, we have $\text{SE}_{\hat{p}} \doteq 0.01155$, so the margin of error for 95% confidence is $1.96\text{SE}_{\hat{p}} \doteq 0.02263 \doteq 2.26\%$. **(b)** If $\hat{p}$ had been 0.5, then $\text{SE}_{\hat{p}} \doteq 0.01573$ and the margin of error for 95% confidence would be $1.96\text{SE}_{\hat{p}} \doteq 0.03084 \doteq 3.08\%$. **(c)** For samples of about this size, the margin of error is no more than about $\pm 3\%$ no matter what $\hat{p}$ is.

19.28. **(a)** We need to know how they were chosen. (All from the same school? Public or private? Etc.) That they were all in psychology and communications courses makes it seem unlikely that they are truly representative of all undergraduates. **(b)** Using large sample methods, $\hat{p} = \frac{107}{127} \doteq 0.8425$, $\text{SE}_{\hat{p}} \doteq 0.03232$, the margin of error is $2.576\text{SE}_{\hat{p}} \doteq 0.08326$, and the 99% confidence interval is 75.9% to 92.6%. **(c)** With the plus four method, $\tilde{p} = \frac{107+2}{127+4} \doteq 0.8321$, $\text{SE}_{\tilde{p}} \doteq 0.03266$, the margin of error is $2.576\text{SE}_{\tilde{p}} \doteq 0.08413$, and the 99% confidence interval is 74.8% to 91.6%. Both the lower and upper limits of the plus four interval are about 1% lower than those in the large sample interval.

19.29. **(a)** The survey excludes residents of Alaska and Hawaii, and those who have no phones or have only cell phone service. **(b)** Using large sample methods, $\hat{p} = \frac{56}{1128} \doteq 0.0496$, $\text{SE}_{\hat{p}} \doteq 0.00647$, the margin of error is $1.645\text{SE}_{\hat{p}} \doteq 0.01064$, and the 90% confidence interval is 3.9% to 6.0%. **(c)** With plus four methods, $\tilde{p} = \frac{56+2}{1128+4} \doteq 0.0512$, $\text{SE}_{\tilde{p}} \doteq 0.00655$, the margin of error is $1.645\text{SE}_{\tilde{p}} \doteq 0.01078$, and the 90% confidence interval is 4.0% to 6.2%. Both the lower and upper limits of the plus four interval are slightly higher than those in the large sample interval.

19.30. **(a)** For the large-sample interval, we would need at least 15 successes and failures; we have only 8 and 5 failures in the two samples. For plus four intervals, we need only $n \geq 10$ (and confidence level 90% or more). **(b)** For the proportion preferring Times New Roman for Web use: $\tilde{p} = \frac{17+2}{25+4} \doteq 0.6552$, $\text{SE}_{\tilde{p}} \doteq 0.08826$, the margin of error is $1.960\text{SE}_{\tilde{p}} \doteq 0.17299$, and the 95% confidence interval is 0.4822 to 0.8282. For the proportion who prefer Gigi: $\tilde{p} = \frac{20+2}{25+4} \doteq 0.7586$, $\text{SE}_{\tilde{p}} \doteq 0.07946$, the margin of error is $1.645\text{SE}_{\tilde{p}} \doteq 0.13070$, and the 90% confidence interval is 0.6279 to 0.8893.

19.31. **(a)** The failure count (5) is too small for the standard method. The plus four method only requires a confidence level of at least 90% and a sample size of at least 10 (we have $n = 23$). **(b)** Using the plus four method: $\tilde{p} = \frac{18+2}{23+4} \doteq 0.7407$, $\text{SE}_{\tilde{p}} \doteq 0.08434$, the margin of error is $1.645\text{SE}_{\tilde{p}} \doteq 0.1387$, and the 90% confidence interval is 0.6020 to 0.8795.

19.32. **(a)** Both large-sample and plus four methods are safe. For the large-sample interval: $\hat{p} = \frac{171}{880} \doteq 0.1943$, $\text{SE}_{\hat{p}} \doteq 0.01334$, the margin of error is $1.960\text{SE}_{\hat{p}} \doteq 0.02614$, and the 95% confidence interval is 0.1682 to 0.2205. Using plus four methods: $\tilde{p} = \frac{171+2}{880+4} \doteq 0.1957$, $\text{SE}_{\tilde{p}} \doteq 0.01334$, the margin of error is $1.960\text{SE}_{\tilde{p}} \doteq 0.02615$, and the 95% confidence interval is 0.1695 to 0.2219. **(b)** More than 171 respondents have run red lights. We would not expect very many people to claim they *have* run red lights when they have not, but some people will deny running red lights when they have.

19.33. **(a)** Because the smallest number of total tax returns (i.e., the smallest population) is still more than 100 times the sample size, the margin of error will be (approximately) the same for all states. **(b)** Yes, it will change—the sample taken from Wyoming will be about the same size, but the sample from, for example, California will be considerably larger, and therefore the margin of error will decrease.

19.34. **(a)** $n = \left(\frac{2.576}{0.015}\right)^2 (0.2)(0.8) \doteq 4718.8$—so use $n = 4719$. **(b)** $2.576\sqrt{\frac{(0.1)(0.9)}{4719}} \doteq 0.01125$.

19.35. **(a)** The margins of error are $1.96\sqrt{\hat{p}(1-\hat{p})/100} = 0.196\sqrt{\hat{p}(1-\hat{p})}$ (below). **(b)** With $n = 500$, the margins of error are $1.96\sqrt{\hat{p}(1-\hat{p})/500}$. The new margins of error are less than half their former size (in fact, they have decreased by a factor of $\frac{1}{\sqrt{5}} \doteq 0.447$).

	p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(a)	m.e.	.0588	.0784	.0898	.0960	.0980	.0960	.0898	.0784	.0588
(b)	m.e.	.0263	.0351	.0402	.0429	.0438	.0429	.0402	.0351	.0263

19.36. PLAN: We will give a 99% confidence interval for the proportion p of college students who support cracking down on underage drinking.
SOLVE: We have an SRS with a very large sample size, so both large-sample and plus four methods can be used. Using the large-sample method: $\hat{p} = \frac{10010}{14941} \doteq 0.6700$, $SE_{\hat{p}} \doteq 0.00385$, the margin of error is $2.576SE_{\hat{p}} \doteq 0.00991$, and the 99% confidence interval is 0.6601 to 0.6799. With such a large sample, the plus four interval is nearly identical: $\tilde{p} = \frac{10010+2}{14941+4} \doteq 0.6699$, $SE_{\tilde{p}} \doteq 0.00385$, the margin of error is $2.576SE_{\tilde{p}} \doteq 0.00991$, and the 99% confidence interval is 0.6600 to 0.6798.
CONCLUDE: Using either method, we are 99% confident that the proportion of college students who support cracking down on underage drinking is between about 0.66 and 0.68.

19.37. PLAN: We will give a 90% confidence interval for the proportion p of all *Krameria cytisoides* shrubs that will resprout after fire.
SOLVE: We assume the 12 shrubs in the sample can be treated as an SRS. Because the number of resprouting shrubs is just 5, the conditions for a large sample interval are not met. Using the plus four method: $\tilde{p} = \frac{5+2}{12+4} \doteq 0.4375$, $SE_{\tilde{p}} \doteq 0.1240$, the margin of error is $1.645SE_{\tilde{p}} \doteq 0.2040$, and the 90% confidence interval is 0.2335 to 0.6415. (If we construct the large-sample interval despite failing to meet the conditions, we find $\hat{p} = \frac{5}{12} \doteq 0.4167$, $SE_{\hat{p}} \doteq 0.1423$, the margin of error is $1.645SE_{\hat{p}} \doteq 0.2341$, and the 90% confidence interval is 0.1826 to 0.6508.)
CONCLUDE: We are 90% confident that the proportion of *Krameria cytisoides* shrubs that will resprout after fire is between about 0.23 and 0.64.

19.38. PLAN: Let p be the proportion of heterosexuals in high-risk cities with multiple partners who never use condoms. We will test H_0: $p = 1/3$ vs. H_a: $p > 1/3$; we use a one-sided alternative because we are concerned that this proportion might be high.
SOLVE: We were told to consider this group to be an SRS, and the expected counts are easily large enough to make our inference methods safe. With $\hat{p} = \frac{304}{803} \doteq 0.3786$, the test statistic is $z = (0.3786 - 1/3)/\sqrt{\frac{(1/3)(2/3)}{803}} \doteq 2.72$, for which $P = 0.0033$.
CONCLUDE: We have very strong evidence (significant at $\alpha = 0.01$) that more than one-third of this group never use condoms.

19.39. PLAN: We will give a 95% confidence interval for the proportion p of American adults who think that humans developed from earlier species of animals.
SOLVE: We have an SRS with a very large sample size, so both large-sample and plus four methods can be used. Using the large-sample method: $\hat{p} = \frac{594}{1484} \doteq 0.4003$, $SE_{\hat{p}} \doteq 0.01272$, the margin of error is $1.960SE_{\hat{p}} \doteq 0.02493$, and the 95% confidence interval is 0.3753 to 0.4252. With such a large sample, the plus four interval is nearly identical: $\tilde{p} = \frac{594+2}{1484+4} \doteq 0.4005$, $SE_{\tilde{p}} \doteq 0.01270$, the margin of error is $1.960SE_{\tilde{p}} \doteq 0.02490$, and the 95% confidence interval is 0.3756 to 0.4254.
CONCLUDE: Using either method, we are 95% confident that the percent of American adults who think that humans developed from earlier species of animals is between about 37.5% and 42.5%.

19.40. PLAN: We will give a 95% confidence interval for the proportion p of female Hispanic drivers in Boston who wear seat belts.
SOLVE: We have a fairly large SRS from a much larger population, with counts 68 and 49, so both large-sample and plus four methods are safe. For the large-sample interval: $\hat{p} = \frac{68}{117} \doteq 0.5812$, $SE_{\hat{p}} \doteq 0.04561$, the margin of error is $1.960SE_{\hat{p}} \doteq 0.08940$, and the 95% confidence interval is 0.4918 to 0.6706.

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1-PropZInt
 (.4918,.67059)
 p̂=.5811965812
 n=117
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1-PropZInt
 (.49053,.6665)
 p̂=.5785123967
 n=121
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Using plus four methods: $\tilde{p} = \frac{70}{121} \doteq 0.5785$, $SE_{\tilde{p}} \doteq 0.04489$, the margin of error is $1.960SE_{\tilde{p}} \doteq 0.08798$, and the 95% confidence interval is 0.4905 to 0.6665.
CONCLUDE: With either method, we are 95% confident that the proportion of female Hispanic drivers who wear seat belts is between about 0.49 and 0.67.

19.41. PLAN: Let p be the proportion of American adults who think that humans developed from earlier species of animals. To assess the evidence for the claim that less than half of adults hold this belief, we will test H_0: $p = 0.5$ vs. H_a: $p < 0.5$.
SOLVE: We assume this sample can be regarded as an SRS. The expected counts are easily large enough to make our inference methods safe. With $\hat{p} \doteq 0.4003$, the test statistic is $z = (0.4003 - 0.5)/\sqrt{\frac{(0.5)(0.5)}{1484}} \doteq -7.68$, for which $P < 0.0001$.
CONCLUDE: We have very strong evidence that less than half of American adults think that humans developed from earlier species of animals.

19.42. PLAN: Let p be the proportion of Hispanic female drivers who wear seat belts. Because the exercise asks if we are convinced that more than half wear seat belts, we will test H_0: $p = 0.5$ vs. H_a: $p > 0.5$; one might instead choose the two-sided alternative $p \neq 0.5$.
SOLVE: Large-sample inference procedures can be used: we view our sample as an SRS, and the expected counts np_0 and $n(1 - p_0)$ are both 58.5. With $\hat{p} \doteq 0.5812$, the test statistic is $z = (0.5812 - 0.5)/\sqrt{\frac{(0.5)(0.5)}{117}} \doteq 1.76$. The P-value is 0.0392 (from Table A) or 0.0395 (software).
CONCLUDE: We have pretty strong evidence—significant at $\alpha = 0.05$, though not at $\alpha = 0.01$—that a majority of Hispanic female drivers in Boston wear seat belts. (If we had chosen a two-sided alternative, the P-value would be twice as big, and the evidence would not be significant at $\alpha = 0.05$.)

19.43. PLAN: We will give a 95% confidence interval for the proportion p of all studies which discuss the success of blinding.

SOLVE: We have a random sample of 97 studies; we assume this can be regarded as an SRS. Because the number of "successes" is just 7, the conditions for a large sample interval are not met. Using the plus four method: $\tilde{p} = \frac{7+2}{97+4} \doteq 0.0891$, $SE_{\tilde{p}} \doteq 0.02835$, the margin of error is $1.960 SE_{\tilde{p}} \doteq 0.05556$, and the 95% confidence interval is 0.0335 to 0.1447. (If we construct the large-sample interval despite failing to meet the conditions, we find $\hat{p} = \frac{7}{97} \doteq 0.0722$, $SE_{\hat{p}} \doteq 0.02627$, the margin of error is $1.960 SE_{\hat{p}} \doteq 0.05149$, and the 95% confidence interval is 0.0207 to 0.1237.)

CONCLUDE: We are 95% confident that the proportion of studies which discuss the success of blinding is between about 0.0335 and 0.1447.

19.44. STATE: For estimating seat belt usage among Hispanic females with 95% confidence, how large a sample is needed to reduce the margin of error to ±0.05?

PLAN: We seek n so that $z^*\sqrt{\frac{p^*(1-p^*)}{n}} = 0.05$, where p^* is our preliminary estimate of the unknown proportion p.

SOLVE: We estimate p using the value of $\hat{p}$ found in the first sample: $p^* = \hat{p} = \frac{68}{117} \doteq 0.5812$. To achieve the desired margin of error, we need $n = \left(\frac{1.96}{0.05}\right)^2 p^*(1-p^*) \doteq 374.03$.

CONCLUDE: A sample size of $n = 375$ Hispanic female drivers will result in a margin of error of approximately ±0.05 for estimating seat belt usage.