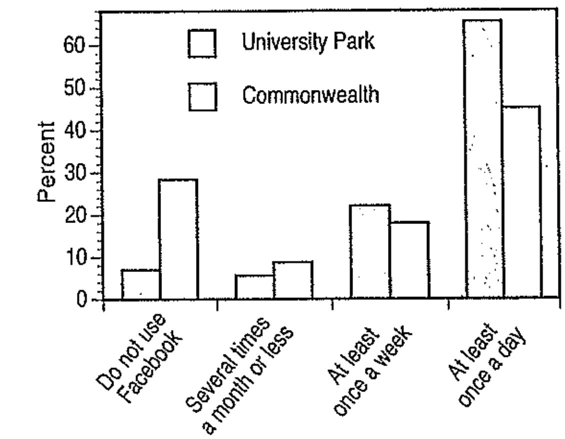
# Chapter 22 Solutions

**22.1.** (a) Note that the two columns of numbers represent (respectively) 978 and 875 students. To find the conditional distributions (shown in the table on the right), divide each count by its column total; for example, the percent of University Park students who do not use Facebook is $\frac{68}{978} \doteq 7.0\%$.
(b) The bar graph (right) reveals that students on the main campus are much more likely to use Facebook at least daily, while commonwealth campus students are more likely not to use it at all.

| | Univ. Park | Common-wealth |
|---|---|---|
| Do not use Facebook | 7.0% | 28.3% |
| Several times a month or less | 5.6% | 8.7% |
| At least once a week | 22.0% | 17.9% |
| At least once a day | 65.4% | 45.0% |

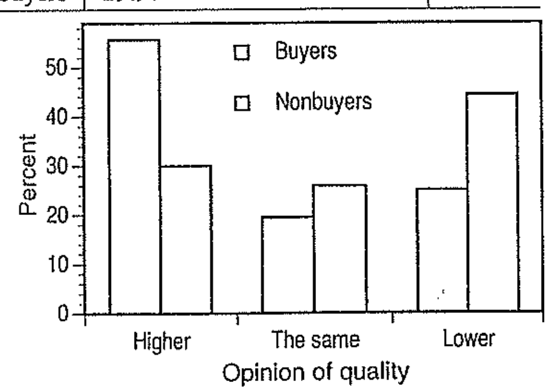



**Note:** *In this and similar problems, some students may not realize that they need to compute "marginal" totals first, and instead might take ratios of numbers that appear in the table. It may help to emphasize that proportions should take the form "part over whole," and that the "whole" sometimes needs to be found by putting all the "parts" together.*

*Furthermore, some might compute the* wrong *marginal totals—for example,* $\frac{68}{68+248}$. *To determine the proper ratios, a good practice is to identify—if possible—which variable is explanatory (as we did for scatterplots). In this case, campus is explanatory, so we should compute percents for each level of that variable, by adding up the counts in each column.*

**22.2.** (a) Compare the distributions of opinion among buyers and nonbuyers. That is, find each count as a percent of its row total; for example, $\frac{20}{20+7+9} = \frac{20}{36} \doteq 55.6\%$ of those who buy recycled filters believe the quality is higher. Buyers are more likely to say "higher" and less likely to say "lower." (b) It may be that actual use convinces people that the recycled filters are high quality. Or it may be that people use recycled filters because they think in advance that their quality is high.

| | *Think the quality is:* Higher | The same | Lower | TOTAL |
|---|---|---|---|---|
| Buyers | 55.6% | 19.4% | 25.0% | 100% |
| Nonbuyers | 29.9% | 25.8% | 44.3% | 100% |



**22.3.** **(a)** To test $H_0$: $p_1 = p_2$ vs. $H_a$: $p_1 \neq p_2$ for the proportions not using Facebook, we have $\hat{p}_1 = \frac{68}{978} \doteq 0.0695$ and $\hat{p}_2 = \frac{248}{875} \doteq 0.2834$. The pooled proportion is $\hat{p} \doteq 0.1705$ and the standard error is $SE \doteq 0.01750$, so $z \doteq -12.22$, for which $P$ is very small. **(b)** To test $H_0$: $p_1 = p_2$ vs. $H_a$: $p_1 \neq p_2$ for the proportions who use Facebook at least weekly, we have $\hat{p}_1 = \frac{215}{978} \doteq 0.2198$ and $\hat{p}_2 = \frac{157}{875} \doteq 0.1794$. The pooled proportion is $\hat{p} \doteq 0.2008$ and the standard error is $SE \doteq 0.01864$, so $z \doteq 2.17$, for which $P = 0.0300$. **(c)** If we did four individual tests, we would not know how confident we could be in all four results taken together.

**22.4.** **(a)** Either large-sample or plus four methods could be used. Both are summarized in the table below.

| | | Standard error | Margin of error | Confidence interval |
|---|---|---|---|---|
| Associate's degree | $\hat{p} = \frac{169}{234} \doteq 0.7222$ | 0.02928 | 0.05739 | 0.6648 to 0.7796 |
| | $\tilde{p} = \frac{169+2}{234+4} \doteq 0.7185$ | 0.02915 | 0.05714 | 0.6614 to 0.7756 |
| Bachelor's degree | $\hat{p} = \frac{256}{321} \doteq 0.7975$ | 0.02243 | 0.04396 | 0.7535 to 0.8415 |
| | $\tilde{p} = \frac{256+2}{321+4} \doteq 0.7938$ | 0.02244 | 0.04398 | 0.7499 to 0.8378 |
| Master's degree | $\hat{p} = \frac{114}{132} \doteq 0.8636$ | 0.02987 | 0.05854 | 0.8051 to 0.9222 |
| | $\tilde{p} = \frac{114+2}{132+4} \doteq 0.8529$ | 0.03037 | 0.05952 | 0.7934 to 0.9125 |

**(b)** The 95% confidence level only applies to each individual interval.
**Note:** *Collectively, the three intervals have confidence level* $0.95^3 \doteq 85.7\%$.

**22.5.** **(a)** Expected counts are below observed counts in the table on the right. For example, $\frac{(131)(627)}{1537} \doteq 53.44$. The expected counts add up to the same values as the observed counts. **(b)** Commonwealth students use Facebook less than weekly more often than we would expect, and use it daily less often than we expect.

**Minitab output**

| | UPark | Cwlth | Total |
|---|---|---|---|
| Monthly | 55 | 76 | 131 |
| | 77.56 | 53.44 | |
| Weekly | 215 | 157 | 372 |
| | 220.25 | 151.75 | |
| Daily | 640 | 394 | 1034 |
| | 612.19 | 421.81 | |
| Total | 910 | 627 | 1537 |

**22.6.** **(a)** The expected counts are shown in the Minitab output on the right; for example, $\frac{(36)(49)}{133} \doteq 13.26$. It is easy to confirm that the expected counts and observed counts give the same row and column totals. **(b)** The largest differences between observed and expected counts are in the "higher" and "lower" columns. These differences are consistent with the observation made in Exercise 22.2: buyers are more likely to say "higher" and less likely to say "lower."

**Minitab output**

| | Higher | Same | Lower | Total |
|---|---|---|---|---|
| Buyers | 20 | 7 | 9 | 36 |
| | 13.26 | 8.66 | 14.08 | |
| Non | 29 | 25 | 43 | 97 |
| | 35.74 | 23.34 | 37.92 | |
| Total | 49 | 32 | 52 | 133 |

**22.7.** **(a)** All expected counts are well above 5 (the smallest is 53.44). **(b)** We test $H_0$: there is no relationship between setting and Facebook use vs. $H_a$: there is some relationship. Reading from the Minitab output, we have $\chi^2 = 19.489$ and $P < 0.0005$. **(c)** The largest contributions come from the first row, reflecting the fact that monthly use is lower among University Park students, and higher among commonwealth students.

**22.8.** **(a)** The smallest expected count is 8.66—slightly more than 5. **(b)** The test statistic is $\chi^2 \doteq 7.638$, and the $P$-value is 0.022. Rejecting $H_0$ means that we have evidence that buyers and nonbuyers of recycled coffee filters have different opinions about the quality of those products. **(c)** The biggest discrepancies between observed and expected counts are those in the first and third columns, which again confirms the relationship observed in Exercises 22.2 and 22.6: buyers are more likely to say "higher" and less likely to say "lower."

**22.9.** PLAN: We test the null hypothesis that there is no relationship between education and belief in astrology. The alternative hypothesis is that there is some relationship.
SOLVE: We have data from the GSS's random sample, and the expected counts are all greater than 5. Reading from the Minitab output, we have $\chi^2 = 10.582$, df $= 2$, and $P = 0.005$. The primary contributions to $\chi^2$ come from the first and third entries on the second row.
CONCLUDE: We have strong evidence that there is a relationship. Specifically, those with associate's degrees are most likely—and those with master's degrees are least likely—to accept astrology as science.

**22.10.** PLAN: We test $H_0$: there is no relationship between age and reliance on a cell phone vs. $H_a$: there is some relationship.
SOLVE: Minitab output is shown on the right; all expected counts are greater than 5. We have $\chi^2 = 127.385$, df $= 3$, and $P < 0.0005$. The primary contributions to $\chi^2$ come from the first and last rows.
CONCLUDE: We have strong evidence that there is a relationship. Specifically, the table confirms our suspicion: about 47% of the youngest age group rely entirely on a cell phone, while that proportion drops to about 21% for the next age group, then to 11.4%, and only 4.3% for the over-65 group.

**Minitab output**

```
          Landline CellOnly    Total
Age1829        108       96      204
            161.14    42.86

Age3049        264       70      334
            263.83    70.17

Age5064        202       26      228
            180.10    47.90

Age65up        178        8      186
            146.92    39.08

Total          752      200      952

ChiSq =     17.526 + 65.897 +
             0.000 +  0.000 +
             2.663 + 10.012 +
             6.573 + 24.713 = 127.38
df = 3, p = 0.000
```

**22.11.** **(a)** df $= (r-1)(c-1) = (3-1)(2-1) = 2$. **(b)** The largest critical value shown for df $= 2$ is 15.20; since the computed value (19.489) is greater than this, we conclude that $P < 0.0005$. **(c)** With $r = 4$ and $c = 2$, the appropriate degrees of freedom would be df $= 3$

**22.12.** **(a)** df $= (r-1)(c-1) = (2-1)(3-1) = 2$. **(b)** On the df $= 2$ row of Table D, we find that $7.38 < \chi^2 < 7.82$, so $0.02 < P < 0.025$, which is consistent with Minitab's reported value, $P = 0.022$. **(c)** If $H_0$ is true, the mean value of $\chi^2$ would be 2 (the degrees of freedom). The observed value is quite a bit larger than this.

**22.13.** We test $H_0$: $p_1 = p_2 = p_3 = \frac{1}{3}$ vs. $H_a$: not all three are equally likely. There were 53 bird strikes in all, so the expected counts are each $53 \times \frac{1}{3} \doteq 17.67$. The chi-square statistic is

$$\chi^2 = \sum \frac{(\text{observed count} - 17.67)^2}{17.67} = \frac{(31-17.67)^2}{17.67} + \frac{(14-17.67)^2}{17.67} + \frac{(8-17.67)^2}{17.67}$$
$$= 10.06 + 0.76 + 5.29 = 16.11.$$

The degrees of freedom are df $= 2$. From Table D, we see that $\chi^2 = 16.11$ falls beyond the 0.0005 critical value. So $P < 0.0005$ and there is very strong evidence that the three tilts differ. The data and the terms of chi-square show that more birds than expected strike the vertical window and fewer than expected strike the 40 degree window.

**22.14.** **(a)** If all days were equally likely, we would have $p_1 = p_2 = \cdots = p_7 = \frac{1}{7}$, and would expect 100 births on each day. **(b)** The chi-square statistic is $\chi^2 = 19.12$, computed as

$$\frac{(84-100)^2}{100} + \frac{(110-100)^2}{100} + \frac{(124-100)^2}{100} + \frac{(104-100)^2}{100} + \frac{(94-100)^2}{100} + \frac{(112-100)^2}{100} + \frac{(72-100)^2}{100}.$$

**(c)** We have df $= 7 - 1 = 6$, so we see that $0.0025 < P < 0.005$ (software gives 0.004); we have strong evidence that births are not spread evenly across the week.

**22.15.** The details of the computation are shown below. The expected counts are found by multiplying the expected frequencies by 803 (the total number of observations).

| | Expected Frequency | Observed Count | Expected Count | $O - E$ | $\frac{(O-E)^2}{E}$ |
|---|---|---|---|---|---|
| 16 to 29 | 0.328 | 401 | 263.384 | 137.616 | 71.9032 |
| 30 to 59 | 0.594 | 382 | 476.982 | −94.982 | 18.9139 |
| 60 or older | 0.078 | 20 | 62.634 | −42.634 | 29.0203 |
| | | 803 | | | 119.8374 |

The difference is significant: $\chi^2 \doteq 119.84$, df $= 2$, $P$ is very small. The largest contribution comes from the youngest age group, which is cited more frequently than we would expect. The other two age groups, which are cited less frequently than expected, also have large contributions. (Any one of the three contributions would be significant by itself.)

**22.16.** **(a)** See the table on the right; for example, $\frac{22}{91} \doteq 24.2\%$ received A's. (There were 91 students in the class.) **(b)** Expected counts are also given in the table; for example, $(91)(0.32) = 29.12$. **(c)** We test $H_0$: $p_1 = 0.32$, $p_2 = 0.41$, $p_3 = 0.20$, $p_4 = 0.07$ vs. $H_a$: at least one of these probabilities is different. (Of course, if one value of $p_i$ is different from those listed in $H_0$, then at least one more must be different!) The guideline for using chi-square is satisfied: all the expected counts are greater than 5. The chi-square statistic is

| | A | B | C | D/F |
|---|---|---|---|---|
| Percent | 24.2% | 41.8% | 22.0% | 12.1% |
| Exp. count | 29.12 | 37.31 | 18.20 | 6.37 |

$$\frac{(22-29.12)^2}{29.12} + \frac{(38-37.31)^2}{37.31} + \frac{(20-18.20)^2}{18.20} + \frac{(11-6.37)^2}{6.37} \doteq 5.297.$$

We have df $= 4 - 1 = 3$, so we see that $0.15 < P < 0.20$ (software gives 0.1513); there is not enough evidence to conclude that the professor's grade distribution was different from the TA grade distribution.

**22.17.** STATE: Does the GSS data suggest that births are not spread uniformly across the year?
PLAN: We test $H_0$: $p_1 = p_2 = \cdots = p_{12} = \frac{1}{12}$ vs. $H_a$: at least one $p_i$ is not $\frac{1}{12}$.
SOLVE: There were 4344 responses, so we would expect $\frac{4344}{12} = 362$ in each group. The $X^2$ statistic is

$$\frac{(321-362)^2}{362} + \frac{(360-362)^2}{362} + \frac{(367-362)^2}{362} + \cdots \frac{(355-362)^2}{362} \doteq 19.76.$$

With df $= 11$, we see from Table E that $0.025 < P < 0.05$ (software gives 0.0487).
CONCLUDE: We have fairly good evidence (significant at $\alpha = 0.05$) that births are not uniformly spread through the year. The largest contributions to the chi-square statistic were from five signs (Aries, Virgo, Scorpio, Sagittarius, and Libra), three with lower-than-expected counts, and two with higher-than-expected counts.

**Note:** *Because of the large sample size, statistical significance was almost a foregone conclusion, and in this case is not indicative of a sharp deviation from $H_0$. The smallest and largest of the 12 observed proportions (0.0739 and 0.0925) are not very different from $\frac{1}{12} = 0.08\overline{3}$).*

**22.18.** (b) The numbers in the first (female) column add to 2625.

**22.19.** (a) This fraction is $\frac{1174}{2625} \doteq 44.7\%$.

**22.20.** (a) The corresponding fraction of males is $\frac{756}{2252} \doteq 33.6\%$.

**22.21.** (c) The expected count is $\frac{(1930)(2625)}{4877} \doteq 1038.8$.

**22.22.** (a) This term in the chi-square statistic is $\frac{(1174-1038.8)^2}{1038.8} \doteq 17.6$.

**22.23.** (a) The degrees of freedom are df $= (r-1)(c-1) = (5-1)(2-1) = 4$.

**22.24.** (b) The null hypothesis of this chi-square test says that gender is not related to opinion about marriage.

**22.25.** (a) Alternatives for such tests are "many-sided"; they do not specify any direction for the difference in the distributions.

**22.26.** (c) $\chi^2 = 69.8$ is larger than the last critical value on the df = 4 line (20.00, for $p = 0.0005$).

**22.27.** (b) While a large sample and large cell counts are nice to have, the most important issue is that we have an SRS (or something close to it).

**22.28.** (a) The sample proportions are $\hat{p}_u = \frac{300}{576} \doteq 0.5208$ and $\hat{p}_r = \frac{174}{561} \doteq 0.3102$. The standard error is SE $\doteq 0.02854$, so the large-sample 95% confidence interval for $p_u - p_r$ is

$$\begin{aligned}\hat{p}_u - \hat{p}_r &\pm 1.96\,\text{SE}\\ &\doteq 0.2107 \pm 0.05594\\ &\doteq 0.1547 \text{ to } 0.2666.\end{aligned}$$

**Minitab output**

| | Urban | Suburban | Rural | Total |
|---|---|---|---|---|
| Brdband | 300 | 521 | 174 | 995 |
| | 260.51 | 480.77 | 253.73 | |
| NoBroad | 276 | 542 | 387 | 1205 |
| | 315.49 | 582.23 | 307.27 | |
| Total | 576 | 1063 | 561 | 2200 |

```
ChiSq =  5.986 +  3.367 + 25.051 +
         4.943 +  2.780 + 20.685 = 62.813
df = 2, p = 0.000
```

Alternatively, use the plus four method: $\tilde{p}_u = \frac{300+1}{576+2} \doteq 0.5208$ and $\tilde{p}_r = \frac{174+1}{561+2} \doteq 0.3108$, the standard error is SE $\doteq 0.02850$, and the interval is

$$\tilde{p}_r - \tilde{p}_s \pm 1.96\,\text{SE} \doteq 0.2099 \pm 0.05586 \doteq 0.1541 \text{ to } 0.2658.$$

**(b)** Along with $\hat{p}_u \doteq 0.5208$ and $\hat{p}_r \doteq 0.3102$ found in part (a), $\hat{p}_s = \frac{521}{1063} \doteq 0.4901$. Overall, the proportion is lowest in rural communities and highest in urban communities (although the difference between the urban and suburban proportions is very small). To determine whether the relationship is significant, we test $H_0$: $p_r = p_s = p_u$ vs. $H_a$: some proportion is different. All expected counts are much more than 5, so the guidelines for the chi-square test are satisfied. We find $\chi^2 = 62.813$, df = 2, and $P < 0.0005$, so the evidence for the observed relationship is very strong.

**22.29.** (a) Out of 1977 adults, 1237 would allow a racist to speak, so the sample proportion is $\hat{p} = \frac{1237}{1977} \doteq 0.6257$, the standard error is SE $\doteq 0.01088$, and the large-sample 99% confidence interval is

$$\begin{aligned}\hat{p} \pm 2.576\,\text{SE} &\doteq 0.6257 \pm 0.02804\\ &\doteq 0.5977 \text{ to } 0.6537.\end{aligned}$$

**Minitab output**

| | Black | White | Other | Total |
|---|---|---|---|---|
| Allow | 140 | 976 | 121 | 1237 |
| | 168.31 | 911.01 | 157.68 | |
| Not | 129 | 480 | 131 | 740 |
| | 100.69 | 544.99 | 94.32 | |
| Total | 269 | 1456 | 252 | 1977 |

```
ChiSq =  4.762 +  4.636 +  8.531 +
         7.961 +  7.749 + 14.260 = 47.899
df = 2, p = 0.000
```

With the plus four method: $\tilde{p} = \frac{1237+2}{1977+4} \doteq 0.6254$, SE $\doteq 0.01087$, and the interval is

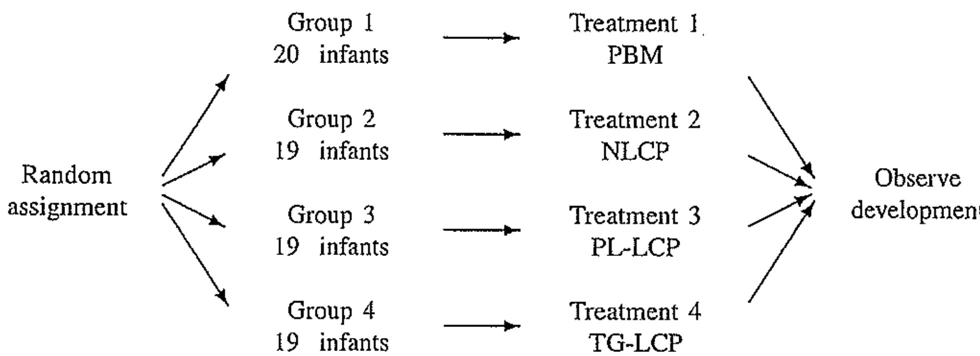
$$\tilde{p} \pm 2.576\,\text{SE} \doteq 0.6254 \pm 0.02801 \doteq 0.5974 \text{ to } 0.6535.$$

**(b)** These percents are $\hat{p}_b = \frac{140}{269} \doteq 0.5204 \doteq 52.0\%$, $\hat{p}_w = \frac{976}{1456} \doteq 0.6703 \doteq 67.0\%$, and $\hat{p}_o = \frac{121}{252} \doteq 0.4802 \doteq 48.0\%$. The proportion of whites is noticeably higher than the other two proportions. We test $H_0$: $p_b = p_w = p_o$ vs. $H_a$: some proportion is different; all expected counts are large enough to use the chi-square test. We find $\chi^2 \doteq 47.899$, df = 2, and $P < 0.0005$; there is very strong evidence that attitudes differ.

**22.30.** We test $H_0$: all proportions are equal vs. $H_a$: some proportions are different. To find the entries in the table (right), take (0.21)(800), (0.25)(800), and (0.28)(800). We find $\chi^2 = 10.619$ with df = 2, so $P < 0.005$—strong evidence that the contact method makes a difference in response.

| | Yes | No |
|---|---|---|
| Phone | 168 | 632 |
| One-on-one | 200 | 600 |
| Anonymous | 224 | 576 |

**22.31.** **(a)** The diagram is shown below. To perform the randomization, label the infants 01 to 77, and choose pairs of random digits. **(b)** See the Minitab output (below, left) for the two-way table. We find $\chi^2 = 0.568$, df = 3, and $P = 0.904$. There is no reason to doubt that the randomization "worked."




**Minitab output for Exercise 22.31**

| | Female | Male | Total |
|---|---|---|---|
| PBM | 11 | 9 | 20 |
| | 10.91 | 9.09 | |
| NLCP | 11 | 8 | 19 |
| | 10.36 | 8.64 | |
| PL-LCP | 11 | 8 | 19 |
| | 10.36 | 8.64 | |
| TG-LCP | 9 | 10 | 19 |
| | 10.36 | 8.64 | |
| Total | 42 | 35 | 77 |

```
ChiSq =  0.001 +  0.001 +
         0.039 +  0.047 +
         0.039 +  0.047 +
         0.179 +  0.215 = 0.568
df = 3, p = 0.904
```

**Minitab output for Exercise 22.32**

| | Favor | Oppose | Total |
|---|---|---|---|
| HighSch | 1010 | 369 | 1379 |
| | 973.28 | 405.72 | |
| Bachelor | 319 | 185 | 504 |
| | 355.72 | 148.28 | |
| Total | 1329 | 554 | 1883 |

```
ChiSq =   1.385 +  3.323 +
          3.790 +  9.092 = 17.590
df = 1, p = 0.000
```

**22.32.** **(a)** To test $H_0$: $p_h = p_b$ vs. $H_a$: $p_h \neq p_b$, we find sample proportions $\hat{p}_h = \frac{1010}{1379} \doteq 0.7324$ and $\hat{p}_b = \frac{319}{504} \doteq 0.6329$, pooled proportion $\hat{p} = \frac{1010+139}{1379+504} \doteq 0.7058$, standard error SE $\doteq 0.02372$, and test statistic $z = (\hat{p}_h - \hat{p}_b)/\text{SE} \doteq 4.19$. The two-sided $P$-value is $2P(Z > 4.19) < 0.00005$. We have very strong evidence of a difference in the proportions favoring the death penalty. **(b)** The chi-square statistic is $\chi^2 = 17.590$. For df = 1, Table D tells us that $P < 0.0005$, which is confirmed by the Minitab output (above, right). **(c)** Both the test statistics ($\chi^2 = 17.590 \doteq 4.19^2 = z^2$) and the $P$-values agree, up to rounding.

**22.33.** **(a)** We test $H_0$: $p_1 = p_2$ vs. $H_a$: $p_1 < p_2$. **(b)** The $z$ test must be used because the chi-square procedure will not work for a one-sided alternative. The sample proportions are $\hat{p}_1 = \frac{11}{30} \doteq 0.3667$ and $\hat{p}_2 = \frac{22}{30} \doteq 0.7333$, and the pooled proportion is $\hat{p} = \frac{33}{60} = 0.55$. Then SE $\doteq 0.12845$, so $z = (\hat{p}_1 - \hat{p}_2)/\text{SE} \doteq -2.85$. This gives $P = 0.0022$, so we reject $H_0$; there is strong evidence that attitude influences tumor growth.

| | Tumor | No tumor |
|---|---|---|
| Group 1 | 11 | 19 |
| Group 2 | 22 | 8 |

**22.34.** STATE: Is there a difference between how men and women assess their chances of being rich by age 30?
PLAN: We test $H_0$: there is no relationship between gender and self-assessment of chances of being rich vs. $H_a$: there is some relationship.
SOLVE: The Minitab output shows that the differences between men and women are highly significant: $\chi^2 = 43.946$, df $= 4$, $P < 0.0005$.
CONCLUDE: Overall, men give themselves a better chance of being rich. This difference shows up most noticeably in the second and fifth rows of the table: women were more likely to say, "some, but probably not," while men more often responded, "almost certain." There was virtually no difference between men and women in the "almost no chance" and "a 50-50 chance" responses, and little difference in "a good chance."

The impact of these differences can be seen in the expected values and chi-square terms: the "some, but probably not" terms account for over 75% of the $\chi^2$ value, and "almost certain" accounts for another 17%. The first and third rows add only 0.021 to the total.

**22.35.** STATE: Does ad sexual content differ in magazines aimed at different audiences?
PLAN: We test $H_0$: there is no relationship between ad sexual content and magazine audience vs. $H_a$: there is some relationship.
SOLVE: The Minitab output shows that the observed differences in sexual content are highly significant: $\chi^2 = 80.874$, df $= 2$, $P < 0.005$.
CONCLUDE: Magazines aimed at women are much more likely to have sexual depictions of models than the other two types of magazines. Specifically, about 39% of ads in women's magazines show sexual depictions of models, compared to 21% and 17% of ads in general-audience and men's magazines. The two women's chi-squared terms account for over half of the total.

**22.36.** **(a)** The best choice is to compute the percents across each row (as in the table on the right). Note, however, that because there were 24 children in each group, we could get the same impression by comparing the raw counts. **(b)** Six cells have expected counts below 5; in fact, two cells have expected counts below 1. **(c)** Most statistical software should give a warning for this analysis.

| | Correct answers | | | | |
|---|---|---|---|---|---|
| Teacher | 0 | 1 | 2 | 3 | 4 |
| Knowledgeable | 20.8% | 4.2% | 25.0% | 12.5% | 37.5% |
| Ignorant | 83.3% | 0.0% | 12.5% | 0.0% | 4.2% |

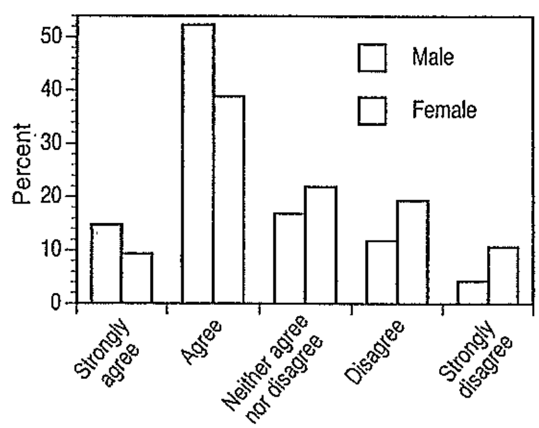
**22.37.** We need cell counts, not just percents. (If we had at least been given the number of travelers in each group—leisure and business—we could estimate the counts.)

**22.38.** Each respondent could have participated in more than one—or even none—of the categories of Internet use. (There are a total of 3024 responses in the table, so on average, each student lists about 1.6 activities.)

**22.39.** In order to do a chi-square test, each subject can only be counted once. (Each subject was given both treatments, so there are 64 observations in each row of the table.)

**22.40.** **(a)** The appropriate conditional distributions are found by adding up the columns (516 men and 636 women). From the table and bar graph below, we see that men are generally more likely to agree or strongly agree. **(b)** PLAN: We test $H_0$: there is no relationship between gender and opinions about animal testing vs. $H_a$: there is a relationship.
SOLVE: If the null hypothesis were true, the mean of the chi-squared statistic would be 4. The Minitab output below gives $\chi^2 = 47.547$, df $= 4$, and $P < 0.0005$.
**(c)** CONCLUDE: We have highly significant evidence that men and women differ in opinions about animal testing. Specifically, the largest contributions to $\chi^2$ come from the "agree," "disagree," and "strongly disagree" rows of the table (which correspond to the three largest differences in the bar graph).

| | Male | Female |
|---|---|---|
| Strongly agree | 14.73% | 9.28% |
| Agree | 52.33% | 38.84% |
| Neither agree nor disagree | 16.86% | 21.86% |
| Disagree | 11.82% | 19.34% |
| Strongly disagree | 4.26% | 10.69% |

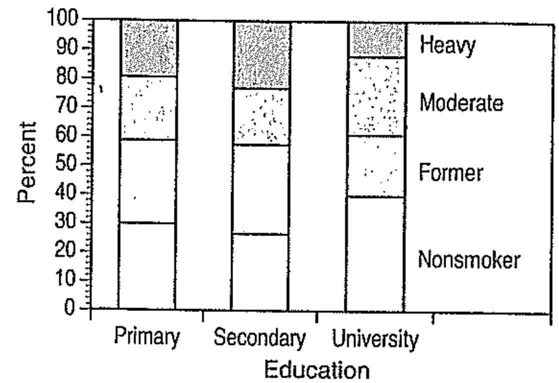



Minitab output

```
            Male    Female    Total
StAgree       76        59      135
           60.47     74.53

Agree        270       247      517
          231.57    285.43

Neither       87       139      226
          101.23    124.77

Disagr        61       123      184
           82.42    101.58

StDisag       22        68       90
           40.31     49.69

Total        516       636     1152

ChiSq =   3.989 +   3.236 +
          6.377 +   5.173 +
          2.000 +   1.623 +
          5.565 +   4.515 +
          8.319 +   6.749 = 47.547
df = 4, p = 0.000
```

**22.41.** (a) The numbers in the first row sum to 187, so the conditional distribution for smoking in the primary-school education group is given in the first row of the table on the right: $\frac{56}{187} \doteq 29.9\%$, $\frac{54}{187} \doteq 28.9\%$, etc. The second and third rows add to 139 and 133, so similar computations give the second and third rows of this table. (Due to rounding, the numbers in the second row add to 99.9%.) Also shown on the right is an example of a graph to display the conditional distributions.

| | *Smoking status* | | | |
|---|---|---|---|---|
| Education | Nonsmoker | Former | Moderate | Heavy |
| Primary | 29.9% | 28.9% | 21.9% | 19.3% |
| Secondary | 26.6% | 30.9% | 19.4% | 23.0% |
| University | 39.8% | 21.1% | 27.1% | 12.0% |




(b) PLAN: We test $H_0$: there is no relationship between education and smoking status vs. $H_a$: there is a relationship.

SOLVE: See the Minitab output below; we find that $\chi^2 = 13.305$, df $= 6$, and $P = 0.039$.

CONCLUDE: The evidence for a relationship is significant at the $\alpha = 0.05$ level. In examining the conditional distributions and the chi-squared details, there is little difference in smoking between the primary and secondary groups—apart from a slight tendency toward heavy smoking among those with a secondary-school education. However, university-educated men seem to be more likely than the other two groups to be nonsmokers or moderate smokers, and less likely to fall in the "former" and "heavy" groups.

**Minitab output**

```
              ns   former moderate    heavy    Total
Primary       56       54       41       36      187
           59.48    50.93    42.37    34.22

Secdry        37       43       27       32      139
           44.21    37.85    31.49    25.44

Univ          53       28       36       16      133
           42.31    36.22    30.14    24.34

Total        146      125      104       84      459

ChiSq =    0.204 +  0.186 +  0.044 +  0.092 +
           1.177 +  0.700 +  0.641 +  1.693 +
           2.704 +  1.866 +  1.141 +  2.858 = 13.305
df = 6, p = 0.039
```

**22.42.** PLAN: To describe the differences, we compare the percents of American and of Asian students who cite each reason. Then we test $H_0$: there is no difference between American and Asian students (all proportions are the same) vs. $H_a$: at least one American/Asian proportion is different.
SOLVE: We compute the percents of each group of students who gave each response by taking each count divided by its column total; for example, $\frac{29}{115} \doteq 25.2\%$:

| | American | Asian |
|---|---|---|
| Save time | 25.2% | 14.5% |
| Easy | 24.3% | 15.9% |
| Low price | 14.8% | 49.3% |
| Live far from stores | 9.6% | 5.8% |
| No pressure to buy | 8.7% | 4.3% |
| Other reason | 17.4% | 10.1% |

Minitab output

```
            Amer   Asian   Total
Save time     29      10      39
           24.37   14.63

Easy          28      11      39
           24.37   14.63

Low price     17      34      51
           31.87   19.13

Live far      11       4      15
            9.37    5.63

No press      10       3      13
            8.12    4.87

Other         20       7      27
           16.88   10.12

Total        115      69     184

ChiSq =    0.878 +  1.463 +
           0.539 +  0.899 +
           6.942 + 11.569 +
           0.282 +  0.469 +
           0.433 +  0.721 +
           0.579 +  0.965 = 25.737
df = 5, p = 0.000
```

Minitab output for the chi-square test is shown on the right; one expected count is less than 5, but this is within our guidelines for using the chi-square test. Note that the chi-square terms for low price account for 18.511 of the total chi-square 25.737. With df = 5, Table D tells us that $P < 0.0005$.
CONCLUDE: There is very strong evidence that Asian and American students buy from catalogs for different reasons; specifically, Asian students place much more emphasis on "low price" and less emphasis on "easy" and "save time."

**22.43.** PLAN: We test the null hypothesis "there is no relationship between race and opinions about schools."
SOLVE: We find $\chi^2 = 22.426$ (df = 8) and $P = 0.004$ (Minitab output at right; all expected cell counts are greater than 5). Nearly half of the total chi-square comes from the first two terms; most of the rest comes from the second and fifth rows.
CONCLUDE: We have strong evidence that there is a relationship; specifically, blacks are less likely and Hispanics are more likely to consider schools excellent, while Hispanics and whites differ in the percent considering schools good (whites are higher) and the percent who "don't know" (Hispanics are higher). Also, a higher percent of blacks rated schools as "fair."

Minitab output

```
          Black    Hisp   White   Total
Exclnt       12      34      22      68
          22.70   22.70   22.59

Good         69      55      81     205
          68.45   68.45   68.11

Fair         75      61      60     196
          65.44   65.44   65.12

Poor         24      24      24      72
          24.04   24.04   23.92

DontKnow     22      28      14      64
          21.37   21.37   21.26

Total       202     202     201     605

ChiSq =  5.047 + 5.620 + 0.015 +
         0.004 + 2.642 + 2.441 +
         1.396 + 0.301 + 0.402 +
         0.000 + 0.000 + 0.000 +
         0.019 + 2.058 + 2.481 = 22.426
df = 8, p = 0.004
```

**22.44.** **(a)** This is not an experiment; no treatment was assigned to the subjects. **(b)** A high nonresponse rate might mean that our attempt to get a random sample was thwarted because of those who did not participate; this nonresponse rate is extraordinarily low. **(c)** PLAN: We perform a chi-square test of the null hypothesis "there is no relationship between olive oil consumption and cancer."

SOLVE: All expected counts are much more than 5, so the chi-square test should be safe. The chi-square statistic is $\chi^2 = 1.552$ (df = 4); if $H_0$ were true, the mean of $\chi^2$ would be 4. This value is smaller than the mean, suggesting that we have little reason to doubt $H_0$. The $P$-value (0.817) confirms this.

CONCLUDE: High olive oil consumption is *not* more common among those without cancer; in fact, when looking at the conditional distributions of olive oil consumption, all percents are between 32.4% and 35.1%—that is, within each group (colon cancer, rectal cancer, control) roughly one-third fall in each olive oil consumption category.

**Minitab output**

```
              Low       Med      High    Total
Colon         398       397       430     1225
           404.39    404.19    416.42

Rectal        250       241       237      728
           240.32    240.20    247.47

Control      1368      1377      1409     4154
          1371.29   1370.61   1412.10

Total        2016      2015      2076     6107

ChiSq =    0.101 +   0.128 +   0.443 +
           0.390 +   0.003 +   0.443 +
           0.008 +   0.030 +   0.007 = 1.552
df = 4, p = 0.817
```

**22.45.** PLAN: We compare how detergent preferences vary by laundry habits, and test the null hypothesis that there is no relationship between laundry habits and preference.

SOLVE: To compare people with different laundry habits, we just compare the percent in each class who prefer the new product.

| | Soft water, warm wash | Soft water, hot wash | Hard water, warm wash | Hard water, hot wash |
|---|---|---|---|---|
| Prefer new product | 54.3% | 51.8% | 61.8% | 58.3% |

The differences are not large, but the "hard water, warm wash" group is most likely to prefer the new detergent. A chi-square test gives $\chi^2 = 2.058$, df $= 3$, and $P = 0.560$.

CONCLUDE: The data give no reason to think that laundry habits influence preference.

**Minitab output**

```
             S/W     S/H     H/W     H/H   Total
Standard      53      27      42      30     152
           49.81   24.05   47.23   30.92

New           63      29      68      42     202
           66.19   31.95   62.77   41.08

Total        116      56     110      72     354

ChiSq =    0.205 + 0.363 + 0.579 + 0.027
           0.154 + 0.273 + 0.436 + 0.020 = 2.058
df = 3, p = 0.560
```

**22.46.** PLAN: We give a 95% confidence interval for $p$, the proportion of American adults who support "other parties."
SOLVE: There are 4483 responses represented in the table; adding the numbers in the bottom row, we find that 65 supported other parties. The counts are large enough to safely use large-sample methods: $\hat{p} = \frac{65}{4483} \doteq 0.0145$ SE $\doteq 0.00179$ and the 95% confidence interval is

$$\hat{p} \pm 0.00350 \doteq 0.0110 \text{ to } 0.0180.$$

The plus four method is (of course) also safe to use: $\tilde{p} = \frac{65+2}{4483+4} \doteq 0.0149$ SE $\doteq 0.00181$ and the 95% confidence interval is

$$\tilde{p} \pm 0.00355 \doteq 0.0114 \text{ to } 0.0185.$$

CONCLUDE: We are 95% confident that the percent of Americans who support other parties is between about 1.1% and 1.8%.

**22.47.** PLAN: We will form a 2 × 5 table of political party and education level, and compute the percents of each education group leaning each direction.

| | None | High school | Jr. College | Bachelor | Graduate |
|---|---|---|---|---|---|
| Democrat | 279 | 996 | 156 | 313 | 218 |
| | 67.4% | 57.7% | 54.7% | 48.2% | 63.0% |
| Republican | 135 | 731 | 129 | 336 | 128 |
| | 32.6% | 42.3% | 45.3% | 51.8% | 37.0% |

SOLVE: Adding up the three "Democrat" rows and the three "Republican" rows gives the counts shown in the accompanying table. Also shown are the percents of each education group leaning in each direction; for example, $\frac{279}{279+135} \doteq 67.4\%$. (It would suffice to compute only one of these percents for each education level.)
CONCLUDE: Of adults who align themselves with either Democrats or Republicans, Democrats are favored by a majority at all education groups except the bachelor's degree level, where Republicans have a slight edge. Support for the Democrats is highest among the least and most educated.

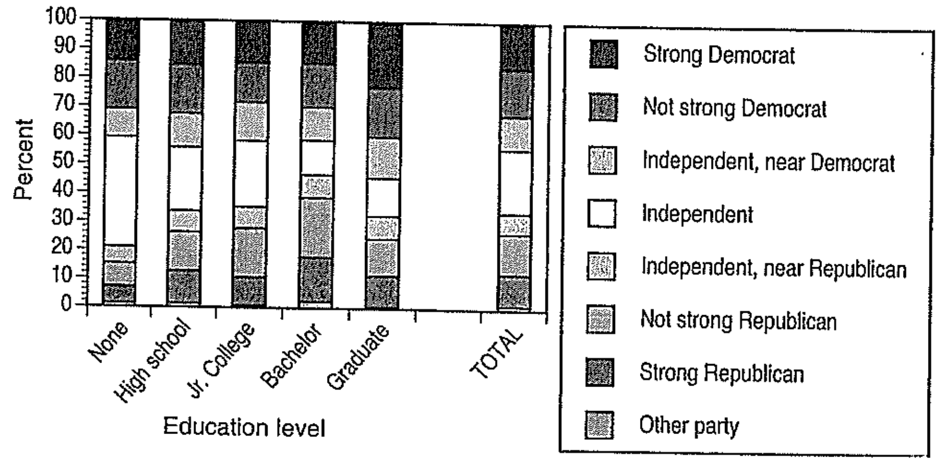
**22.48.** PLAN: We will find conditional distributions for each education level, and perform a chi-square test on the full table, testing the null hypothesis that there is no relationship between education level and political preference.
SOLVE: Shown on the next page are the conditional distributions of political affiliation by education level, and a bar graph displaying these distributions. Highlighted in the table are some numbers that are drastically smaller or larger than the other numbers in their rows. (These also stand out on the bar graph.)

The Minitab output that follows confirms that the differences are highly significant ($\chi^2 = 238.684$, df $= 28$, $P < 0.0005$). We also note that the expected counts are all greater than 5 (although two counts are only *barely* greater than 5), so the test is safe to use.

The largest contributions to the chi-square statistic are highlighted, and match the numbers that stand out in the conditional distributions. Over half of the value of $\chi^2$ comes from the "Independent" row, with a higher-than-expected count of least-educated subjects, and fewer of the bachelor's and graduate degree subjects. The five other double-digit contributions to $\chi^2$ account for over one-third of the total, and arise from the large number of strong Democrats with graduate degrees, and the counts of Republicans and strong Republicans among the least-educated and bachelor's degree groups, which were (respectively) lower and higher than expected.

CONCLUDE: Student observations about the full table will vary, but one thing that stands out is the high percent of Independents in the least-educated group. (This insight was not available in the compressed table because it omitted Independents and supporters of other parties.)

| | None | High school | Junior college | Bachelor | Graduate | TOTAL |
|---|---|---|---|---|---|---|
| Strong Democrat | 14.1% | 15.3% | 14.4% | 14.5% | 22.6% | 15.6% |
| Not strong Democrat | 16.8 | 17.0 | 13.9 | 15.3 | 17.2 | 16.4 |
| Near Democrat | 9.8 | 11.7 | 13.4 | 11.5 | 14.4 | 11.8 |
| Independent | 38.3 | 22.2 | 23.0 | 12.1 | 13.2 | 22.2 |
| Near Republican | 5.7 | 7.4 | 7.5 | 7.9 | 8.0 | 7.3 |
| Not strong Republican | 8.2 | 13.6 | 17.1 | 20.8 | 12.9 | 14.2 |
| Strong Republican | 5.8 | 11.3 | 9.9 | 15.5 | 10.9 | 11.0 |
| Other party | 1.3 | 1.4 | 0.8 | 2.4 | 0.7 | 1.4 |



**Minitab output**

```
            None      HS     JuCo     Bach     Grad    Total
StrongDem     97     347       54      110       91      699
          106.96  352.70    58.31   118.35    62.68

Democrat     115     384       52      116       69      736
          112.62  371.37    61.40   124.61    66.00

NearDem       67     265       50       87       58      527
           80.64  265.91    43.97    89.22    47.26

Indpdnt      263     503       86       92       53      997
          152.56  503.06    83.18   168.80    89.40

NearRep       39     168       28       60       32      327
           50.04  165.00    27.28    55.36    29.32

Repub         56     307       64      158       52      637
           97.48  321.41    53.14   107.85    57.12

StrongRep     40     256       37      118       44      495
           75.75  249.76    41.30    83.81    44.39

Other          9      32        3       18        3       65
            9.95   32.80     5.42    11.00     5.83

Total        686    2262      374      759      402     4483

ChiSq =    0.928 +  0.092 +  0.319 +  0.588 + 12.795 +
           0.050 +  0.430 +  1.440 +  0.595 +  0.136 +
           2.308 +  0.003 +  0.828 +  0.055 +  2.442 +
          79.942 +  0.000 +  0.096 + 34.941 + 14.823 +
           2.435 +  0.055 +  0.019 +  0.388 +  0.244 +
          17.648 +  0.646 +  2.218 + 23.322 +  0.459 +
          16.869 +  0.156 +  0.447 + 13.951 +  0.003 +
           0.090 +  0.019 +  1.082 +  4.446 +  1.373 = 238.684
df = 28, p = 0.000
```