Chapter 18 — Two-Sample Problems

Two-Sample Problems

- goal is to compare responses to 2 treatments or to compare characteristics of 2 populations
- we have separate samples from each treatment/population

☆ unlike matched pairs design, there is no matching of individuals from 2 samples

☆ 2 samples can be different sizes

population	sample size	sample mean	sample s.d.
1	n_1	$\bar{x}_1$	s_1
2	n_2	$\bar{x}_2$	s_2

Conditions for Inference Comparing Two Means

- We have two SRSs from 2 populations
- Samples are independent
- both populations are normally distributed (μ & σ unknown for both populations)

Goal (typically):
To do inference about $\mu_1 - \mu_2$

Two Sample t Procedures

SRS of size n_1 from pop. 1 $\sim N(\mu_1, \sigma_1)$ & SRS of size n_2 from pop. 2 $\sim N(\mu_2, \sigma_2)$

① CI for $\mu_1 - \mu_2$: $$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

☆ choose $df = \min(n_1 - 1, n_2 - 1)$ to use table C.

② To test $H_0: \mu_1 = \mu_2$, calculate 2-sample t-statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Then find pvalues for t w/ $df = \min(n_1 - 1, n_2 - 1)$

Chp 18 (cont)

Ex 1 Identify as ① single sample, ② matched pairs or ③ two independent samples.

Looking back on love. Choose 40 romantically attached couples in their midtwenties. Interview the man and woman separately about a romantic attachment they had at age 15 or 16. Compare the attitudes of men and women.

Whom do you trust? Companies often place advertisements to improve the image of their brand rather than to promote specific products. In a randomized comparative experiment, business students read ads that cited either the *Wall Street Journal* or the *National Enquirer* for important facts about a fictitious company. The students then rated the trustworthiness of each ad on a 7-point scale. Compare the mean score for the two types of advertisement.

Chemical analysis. To check a new analytical method, a chemist obtains a reference specimen of known concentration from the National Institute of Standards and Technology. She then makes 20 measurements of the concentration of this specimen with the new method and checks for bias by comparing the mean result with the known concentration.

Chemical analysis, continued. Another chemist is checking the same new method. He has no reference specimen, but a familiar analytic method is available. He wants to know if the new and old methods agree. He takes a specimen of unknown concentration and measures the concentration 10 times with the new method and 10 times with the old method.

Note: Book refers to Option 2 as

$$df = \min(n_1 - 1, n_2 - 1)$$

and Option 1 as

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$$

which is what software packages use; Option 1 provides smaller CI

* Option 1 approximation accurate when $n_1 \geq 5$ and $n_2 \geq 5$.

(For most of our problems, we'll use option 2 as it's simpler.)

Chp 18 (cont)

Ex 2 **Logging in the rain forest.** "Conservationists have despaired over destruction of tropical rain forest by logging, clearing, and burning." These words begin a report on a statistical study of the effects of logging in Borneo.[4] Here are data on the number of tree species in 12 unlogged forest plots and 9 similar plots logged 8 years earlier:

Unlogged	22	18	22	20	15	21	13	13	19	13	19	15
Logged	17	4	18	14	18	15	15	10	12			

(a) The study report says, "Loggers were unaware that the effects of logging would be assessed." Why is this important? The study report also explains why the plots can be considered to be randomly assigned.

(b) Does logging significantly reduce the mean number of species in a plot after 8 years? Follow the four-step process as illustrated in Examples 18.2 and 18.3.

let population 1 be from plots unlogged.

population 2 from plots logged previously

stemplots:

unlogged

logged

H_0:

H_a:

$\bar{x}_1 =$

$s_1 =$

$n_1 =$

$\bar{x}_2 =$

$s_2 =$

$n_2 =$

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\Rightarrow t = \frac{\bar{x}_1 - \bar{x}_2}{SE} =$$

df =

p-value =

conclusion:

Chap 18 (cont)

Ex 3 **Do good smells bring good business?** Businesses know that customers often respond to background music. Do they also respond to odors? One study of this question took place in a small pizza restaurant in France on Saturday evenings in May. On one of these evenings, a relaxing lavender odor was spread through the restaurant. Table 18.2 gives the time (minutes) that two samples of 30 customers spent in the restaurant and the amount they spent (in euros).[7] The two evenings were comparable in many ways (weather, customer count, and so on), so we are willing to regard the data as independent SRSs from spring Saturday evenings at this restaurant. The authors say, "Therefore at this stage it would be impossible to generalize the results to other restaurants."

(a) Does a lavender odor encourage customers to stay longer in the restaurant? Examine the time data and explain why they are suitable for two-sample t procedures. Use the two-sample t test to answer the question posed.

(b) Does a lavender odor encourage customers to spend more while in the restaurant? Examine the spending data. In what ways do these data deviate from Normality? With 30 observations, the t procedures are nonetheless reasonably accurate. Use the two-sample t test to answer the question posed.

TABLE 18.2 Time (minutes) and spending (euros) by restaurant customers

NO ODOR		LAVENDER	
MINUTES	EUROS SPENT	MINUTES	EUROS SPENT
103	15.9	92	21.9
68	18.5	126	18.5
79	15.9	114	22.3
106	18.5	106	21.9
72	18.5	89	18.5
121	21.9	137	24.9
92	15.9	93	18.5
84	15.9	76	22.5
72	15.9	98	21.5
92	15.9	108	21.9
85	15.9	124	21.5
69	18.5	105	18.5
73	18.5	129	25.5
87	18.5	103	18.5
109	20.5	107	18.5
115	18.5	109	21.9
91	18.5	94	18.5
84	15.9	105	18.5
76	15.9	102	24.9
96	15.9	108	21.9
107	18.5	95	25.9
98	18.5	121	21.9
92	15.9	109	18.5
107	18.5	104	18.5
93	15.9	116	22.8
118	18.5	88	18.5
87	15.9	109	21.9
101	25.5	97	20.7
75	12.9	101	21.9
86	15.9	106	22.5

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Chp 18 (cont)

Ex 3 (cont)

i) (time data)

① Make back-to-back stemplot

(pop. 1) No Scent		(pop 2) Lavendar
	6	
	7	
	8	
	9	
	10	
	11	
	12	
	13	

H_0:

H_a:

time

pop	n	$\bar{x}$	s
1	30	91.2$\bar{6}$	14.9296
2	30	105.7	13.1048

SE =

t =

df =

p-value:

Conclusion:

(b) make back-to-back stemplot (money)

No Scent		Lavendar
	12	
	13	
	14	
	15	
	16	
	17	
	18	
	19	
	20	
	21	
	22	
	23	
	24	
	25	

H_0:

H_a:

money

pop	n	$\bar{x}$	s
1	30	17.5133	2.3588
2	30	21.1233	2.3450

SE =

t =

df =

p-value:

Conclusion:

Chp 18 (cont)

Ex 4

Students' self-concept. A study of the self-concept of seventh-grade students asked if male and female students differ in mean score on the Piers-Harris Children's Self-Concept Scale.[9] Software that uses Option 1 gives these summary results:

Gender	n	Mean	Std dev	Std err	t	df	P
F	31	55.5161	12.6961	2.2803	-0.8276	62.8	0.4110
M	47	57.9149	12.2649	1.7890			

Starting from the sample means and standard deviations, verify each of these entries: the standard errors of the means; the degrees of freedom for two-sample t; the value of t.