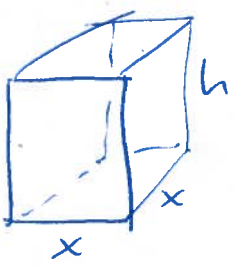
3,3 #57

maximize volume, given cost.



$$\text{Cost} = \$1\,(x^2 + 4xh) + \$5(x^2) = 72$$

(x^2: bottom; $4xh$: sides; $5x^2$: top)

$$x^2 + 4xh + 5x^2 = 72$$

$$6x^2 + 4xh = 72$$

$$4xh = 72 - 6x^2$$

$$h = \frac{72 - 6x^2}{4x} = \frac{36 - 3x^2}{2x}$$

$$V = x^2 h = x^2\left(\frac{36 - 3x^2}{2x}\right) = \frac{x}{2}(36 - 3x^2)$$

$$V = -\frac{3}{2}x^3 + 18x$$

zeros at: $\frac{x}{2} = 0$ or $36 - 3x^2 = 0$

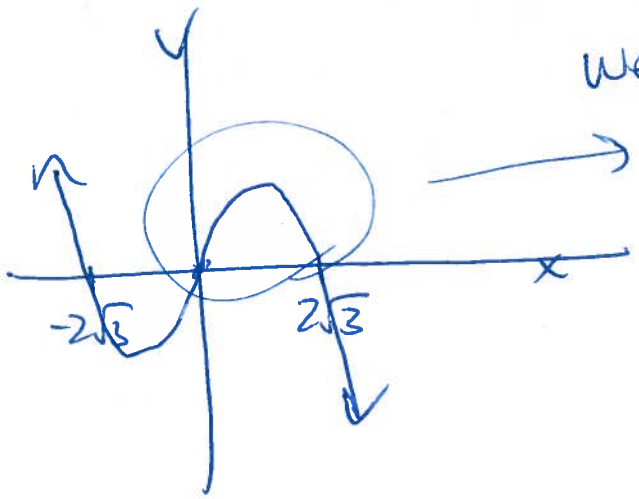
$x = 0$

$3x^2 = 36$

$x^2 = 12 \Rightarrow x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

and we know graph looks something like ↷



We only want to consider x-values where V is positive $\Rightarrow$

$0 \le x \le 2\sqrt{3}$

Guess and check $(2\sqrt{3} \approx 3.464)$

x	$V = \frac{-3}{2}x^3 + 18x$	
1	16.5	
2	24	max
3	13.5	
2.5	21.5625	
1.5	21.9375	

$\Rightarrow$ max volume is $24\ ft^3$ when

$x = 2$ ft and $h = \frac{3L - 3(2^2)}{2(2)} = 6$ ft