

8.4 #43

use

$$\textcircled{1} \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\textcircled{2} \cos(x-y) = \cos x \cos y + \sin x \sin y$$

to prove $2\sin x \sin y = \cos(x-y) - \cos(x+y)$

$$\textcircled{2} - \textcircled{1} \Leftrightarrow \cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$+ (-\cos(x+y) = -\cos x \cos y + \sin x \sin y)$$

$$\cos(x-y) - \cos(x+y) = \sin x \sin y + \sin x \sin y$$

$$= 2\sin x \sin y \quad \checkmark$$

#53 $\textcircled{1} \log x + \log y = 71$

+ $\textcircled{2} 2\log x - \log y = 2$

$$3\log x = 73$$

$$\log x = 73/3$$

$$\boxed{10^{73/3} = x}$$

intersection pt:

$$(10^{73/3}, 10^{140/3})$$

$$\text{or } (\sqrt[3]{10^{73}}, \sqrt[3]{10^{140}})$$

$$\textcircled{1} \log(10^{73/3}) + \log y = 71$$

$$73/3 + \log y = 71$$

$$\log y = 71 - \frac{73}{3}$$

$$\log y = \frac{213 - 73}{3}$$

$$\log y = \frac{140}{3}$$

$$\boxed{y = 10^{140/3}}$$