8.6 (cont)

Ex 4 Find inverse of $A = \begin{bmatrix} 6 & 1 & 20 \\ 1 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$$\begin{array}{c} \\ (-6) \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 6 & 1 & 20 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ -6 & 6 & 0 & 0 & -6 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|ccc} 0 & 7 & 20 & 1 & -6 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ (-7) \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 7 & 20 & 1 & -6 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \\ 0 & -7 & -21 & 0 & 0 & -7 \end{array}\right] \qquad \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & -6 & -7 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ (3) \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & -6 & -7 \\ 0 & 0 & -3 & 3 & -18 & -21 \end{array}\right] \qquad \begin{array}{c} \\ \\ (-1) \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 & -18 & -20 \\ 0 & 0 & -1 & 1 & -6 & -7 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -17 & -20 \\ 0 & 1 & 0 & 3 & -18 & -20 \\ 0 & 0 & 1 & -1 & 6 & 7 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 3 & -17 & -20 \\ 3 & -18 & -20 \\ -1 & 6 & 7 \end{bmatrix}$$

Ex 5 (a) Set up $\begin{aligned} 4x - 7y &= -65 \\ -x + 2y &= 18 \end{aligned}$ as $AX = B$, (b) find A^{-1}, (c) Solve for X.

8.6 (cont)

Ex 6 Use the inverse from Ex 4 to solve

$$6x + y + 20z = 5$$
$$x - y = 2$$
$$y + 3z = -1$$

$A = \begin{bmatrix} 6 & 1 & 20 \\ 1 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

we know from Ex 4, $A^{-1} = \begin{bmatrix} 3 & -17 & -20 \\ 3 & -18 & -20 \\ -1 & 6 & 7 \end{bmatrix}$ and $AX = B$

$\Leftrightarrow A^{-1}AX = A^{-1}B$

$\Leftrightarrow IX = A^{-1}B$

$\Leftrightarrow X = A^{-1}B$

$$\Rightarrow X = \begin{bmatrix} 3 & -17 & -20 \\ 3 & -18 & -20 \\ -1 & 6 & 7 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 - 34 + 20 \\ 15 - 36 + 20 \\ -5 + 12 - 7 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$\Rightarrow x = 1, y = -1, z = 0$ or we can write it as intersection pt $(1, -1, 0)$

Ex 7 If M & N are nonsingular, then MN is nonsingular. Show that the inverse of MN is $N^{-1}M^{-1}$!

M, N nonsingular $\Rightarrow M^{-1}$ exists and N^{-1} exists.

To show MN and $N^{-1}M^{-1}$ are inverses, we need to show $(MN)(N^{-1}M^{-1}) = I$

lets start w/ $(MN)(N^{-1}M^{-1})$

$= MNN^{-1}M^{-1}$

$= M(NN^{-1})M^{-1}$

$= MIM^{-1}$

$= MM^{-1}$

$= I$ ✓